Magnitude determination using initial *P* waves: A single-station approach

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[1] Prompted by recent success in the research and application of earthquake early warning systems, we conducted an investigation into the relationship between the magnitudes of earthquakes and the properties of the first three seconds of the *P* waves at a single station. Using waveforms at station NACB of the Broadband Array in Taiwan for Seismology from 46 earthquakes within 100-km epicentral distance, we found a linear correlation between the magnitudes and the logarithmic characteristic periods of the initial *P* waves for earthquakes up to magnitude 6.5. We also detected a deterministic relationship between the magnitudes and the peak amplitudes of the initial *P* waves for earthquakes of magnitude $M < 6.5$. While further study needs to be done on the behavior of the initial *P*-wave peak amplitudes from larger earthquakes, we propose here that the characteristic periods and peak amplitudes of the first three seconds of the *P* waves be used jointly in onsite (single-station) earthquake early warning operations. **Citation:** Wu, Y.-M., H.-Y. Yen, L. Zhao, B.-S. Huang, and W.-T. Liang (2006), Magnitude determination using initial *P* waves: A single-station approach, *Geophys. Res. Lett.*, *33*, L05306, doi:10.1029/2005GL025395.

1. Introduction

[2] In the past few years, research on earthquake early warning (EEW) has undergone a rapid development [*Wu and Teng*, 2002; *Allen and Kanamori*, 2003; *Kanamori*, 2005; *Wu and Kanamori*, 2005a, 2005b; *Olson and Allen*, 2005]. At present EEW is becoming a useful tool for practical real-time seismic hazard mitigation, with applications in Japan [*Nakamura*, 1988; *Horiuchi et al.*, 2005], Taiwan [*Wu and Teng*, 2002], and Mexico [*Espinosa-Aranda et al.*, 1995].

[3] The critical point of EEW is determining the overall size of an earthquake, and thus the expected strong ground motion, from the first few seconds of the *P* wave. Whether this determination is possible rests on whether there are differences in the onsets of earthquakes of different sizes, which is ultimately controlled by the physics of the earthquake rupture process. A prevalent hypothesis for the earthquake rupture propagation is the so-called "cascade model". In such a model, an earthquake rupture starts with a slip on a small fault patch and continues to grow along the fault plane as long as the conditions are favorable [*Brune*, 1979; *Ellsworth and Beroza*, 1995, 1998; *Kilb and Gomberg*, 1999]. This domino-type concept implies that all earthquakes large and small begin in the same way from a small slip, and therefore the size of an earthquake cannot be determined until the entire rupture has run its course. This scenario appears to contradict the observations on the deterministic relationships between the earthquake magnitudes and some waveform properties, such as the characteristic periods and peak amplitudes, of the first few seconds of *P* waves [*Allen and Kanamori*, 2003; *Kanamori*, 2005; *Wu and Kanamori*, 2005a, 2005b]. As an alternative model for earthquake rupture process, *Olson and Allen* [2005] suggests that the final magnitude of an earthquake is determined by the first few seconds of the rupture process and the state of stress in the region surrounding the fault plane. This model provides a physical basis for the deterministic nature of earthquake magnitudes and for EEW applications.

[4] There are currently two different approaches to the EEW operations. One is the multi-station approach and it has been adopted in previous EEW studies [*Allen and Kanamori*, 2003; *Kanamori*, 2005; *Wu and Kanamori*, 2005a, 2005b] to find the relationships between the final magnitude of an earthquake and the characteristic parameters in the first few seconds of the *P* waves from a number of stations. In the EEW practice, this approach requires a data processing center to collect and analyze the *P*-wave observations and estimate the magnitude. In the present study, however, we investigate an alternative approach that uses the information gathered from the first few seconds of the *P* wave at a single station to estimate the magnitude of an earthquake. This allows the EEW assessments to be done onsite without the need to gather the observations from other stations, thus providing an even faster EEW than the multi-station method.

[5] From a practical point of view, most critical facilities have some automatic "shutdown" provisions whenever large ground acceleration is detected. Therefore, a single-station EEW system will be useful for large earthquakes that occur at a distance beyond about 20 km from the epicenter. However, no EEW system can provide actionable early warning within this distance, because damaging *S*-waves will have arrived before an alarm can be issued.

2. Data and Method

[6] In the single-station or onsite approach, we attempt to find out the relationships between the magnitudes of earthquakes and some observational properties of the first few seconds of the *P* waves, including the characteristic periods τ_c [*Kanamori*, 2005; *Wu and Kanamori*, 2005a] and the peak displacement amplitudes *Pd* [*Wu and Kanamori*, 2005b; Y. M. Wu and L. Zhao, Attenuation relation of *Pd*

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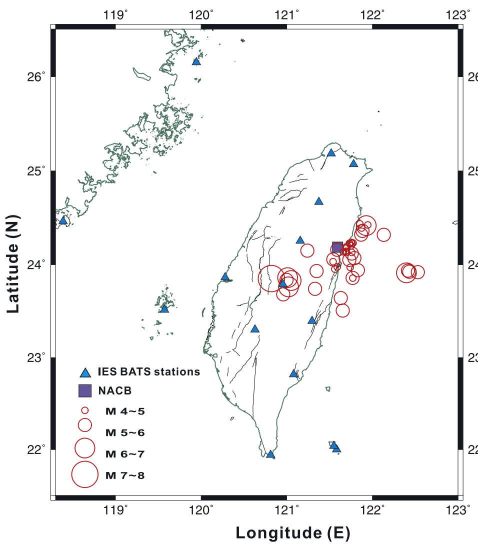

Figure 1. Stations (solid triangles and square) of the Broadband Array in Taiwan for Seismology (BATS) and the epicenters of 46 events (circles) used in this study. The blue solid square shows the location of NACB station.

and its application for earthquake early warning, submitted to *Terrestrial, Atmospheric and Oceanic Sciences*, 2005, hereinafter referred to as Wu and Zhao, submitted manuscript, 2005].

[7] The characteristic period τ_c is defined in terms of the waveforms of the first few seconds of the *P* waves as:

$$\tau_c = 2\pi/\sqrt{r}, \tag{1}$$

with

$$r = \int_0^{\tau_0} \dot{u}^2(t)dt \Big/ \int_0^{\tau_0} u^2(t)dt, \tag{2}$$

where $u(t)$ and $\dot{u}(t)$ are the ground-motion displacement and velocity, respectively, and τ_0 is the duration of the *P* waveform used (usually 3 sec). Previous studies have shown that the characteristic periods reflect the sizes of earthquakes [*Kanamori*, 2005; *Wu and Kanamori*, 2005a; Y. M. Wu et al., Experiment on τ_c and P_d method for earthquake early warning in southern California, submitted to *Bulletin of the Seismological Society of America,* 2005, hereinafter referred to as Wu et al., submitted manuscript, 2005].

[8] The quantity *Pd* is the peak amplitude of displacement within the first few seconds (again usually 3 sec) after the arrival of the *P* wave. In previous studies, we have shown that this peak amplitude is well correlated with peak ground velocity (PGV) in Taiwan [*Wu and Kanamori*, 2005b] and in Southern California (Wu et al., submitted manuscript, 2005). This correlation suggests that we may be able to predict the peak intensity of the overall ground motion using the initial *P* waves. On the other hand, *Pd* is an amplitude parameter and reflects the attenuation relationship of the ground motion with distance. Therefore, if we can determine the attenuation relationship of *Pd*, then we can use *Pd* to estimate the magnitude when the hypocentral distance is available.

[9] In practical implementation of the single-station (onsite) EEW approach, it is important to carefully analyze and choose candidate sites that can provide good EEW signals. We analyzed some of the stations in the Broadband Array in Taiwan for Seismology (BATS) maintained by the Institute of Earth Sciences (IES), Academia Sinica. We found the station NACB to be a good candidate and we will focus on this station in this paper. Figure 1 shows the station distribution of the BATS. All of the BATS stations have both high-gain broad-band velocity and low-gain force-balance acceleration (FBA) sensors. Signals are digitized at 100 samples per second with a 24-bit resolution. The low-gain channels record all large earthquakes without clipping, while the high-gain channels provide the high resolution waveforms for the initial 3 sec of the *P* waves used in this study.

[10] The NACB station is located in a marble rock tunnel in eastern Taiwan, and provides a large number of high-quality waveform records from earthquakes in the surrounding region. The waveforms used in this study were collected from 46 earthquakes occurring between 1996 and 2002. In the Central Weather Bureau (CWB) catalog, all of the events have magnitudes from 4.0 to 7.6 and focal depths of less than 30 km, and all of them are within 100-km epicentral distances from station NACB.

[11] In the CWB earthquake catalog, the reported magnitude is the local magnitude M_L. Taking the saturation problem of the local magnitudes [*Wu et al.*, 2001] into consideration, for events of $M_L \geq 6.5$ we used the Harvard CMT moment magnitude M_w. In this paper, we denote all the earthquake magnitudes simply by *M*.

[12] In determining τ_c and *Pd*, the velocity records were numerically differentiated to obtain the accelerations and integrated to obtain the displacements. We applied a high-pass recursive Butterworth filter with a cutoff frequency of 0.075 Hz to remove the low-frequency drift after numerical integration. An automatic *P* picker described by *Allen* [1978] is used to detect the *P* arrival from the vertical acceleration records. Following the analyses of *Kanamori* [2005], *Wu and Kanamori* [2005a, 2005b] and Wu et al. (submitted manuscript, 2005), we chose a duration of 3 sec for the estimation of τ_c and *Pd*. A good estimate of the characteristic period for large (M > 7) earthquakes will require much longer than 3 seconds of the initial *P*-waves [*Kanamori*, 2005].

3. Results

[13] The result we obtained for the relationship between the characteristic period τ_c and the magnitude *M* is shown in Figures 2a and 2b. Figure 2a is a plot of τ_c determined from all 46 records versus *M*. The values of $\log\tau_c$ increase approximately linearly with *M* with a linear correlation coefficient of 0.77. The linear regression for the relationship between the τ_c and *M* leads to

$$M = 3.088 \log \tau_c + 5.300, \tag{3}$$

with a standard deviation of 0.57. This means that when using τ_c to estimate the magnitude ($M_{\tau c}$), one standard-

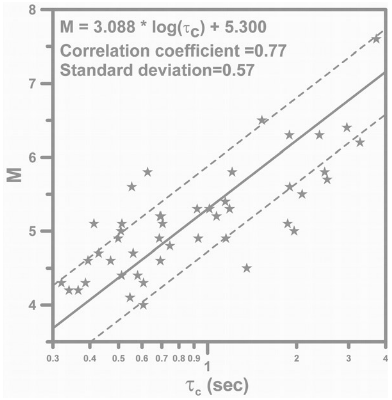

Figure 2a. Relationship between M and logτ_c determined from the 46 events. Solid line shows the least squares fit and two dashed lines show one standard deviation of M.

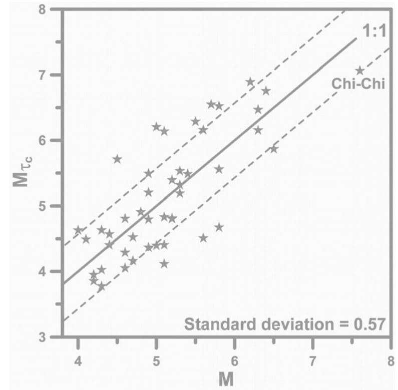

Figure 2b. Magnitudes estimated from τ_c versus M. Solid line shows the least squares fit and the two dashed lines show the range of one standard deviation.

deviation level will be about 0.57 units. This uncertainty is slightly smaller than those in previous studies [*Grecksch and Kumpel*, 1997; *Allen and Kanamori*, 2003]. From Figure 2a, we can see that the linear relationship between logτ_c and M persists throughout the entire magnitude range. This result strongly supports the scenario that the earliest stage of the earthquake rupture probably plays an important role in determining the final magnitude of the earthquake. Not surprisingly, the magnitudes estimated from τ_c correlate very well with the values reported in the earthquake catalogs, as shown in Figure 2b.

[14] We assumed a linear relationship among the logarithmic Pd, the magnitude M, and the logarithmic hypocentral distance (R):

$$\log Pd(R) = A + B \cdot M + C \cdot \log(R) \tag{4}$$

where A, B, and C are constants to be determined from the regression analysis using the P waves from the 46 events. Generally, for such a regression we need to consider the R term. However, we conducted a test for this term and found that it is not statistically significant. Similar result was also found in our previous studies [*Wu et al.*, 2005; Wu and Zhao, submitted manuscript, 2005]. With constraints from the 46 records, the best-fitting attenuation relationship for log Pd is found to be

$$\log Pd(R) = -3.801 + 0.722 \cdot M - 1.444 \cdot \log(R) \pm 0.29. \tag{5}$$

The comparisons between the observed and predicted Pd values for magnitudes 4, 5, 6 and 7 are shown in Figure 3a.

[15] In EEW practice, the earthquake location can also be estimated from the P wave [*Nakamura*, 1988]. The hypocentral distance will then be available, and the magnitude (M_{Pd}) can thus be estimated from Pd. For this purpose, we shall rewrite equation (5) into the following form:

$$M_{Pd} = 5.265 + 1.385 \cdot \log(Pd) + 2.000 \cdot \log(R). \tag{6}$$

Figure 3b provides a comparison of the two types of magnitude values for the 46 events in which the M_{Pd} values are plotted against the M values. On the 45^0 line, $M_{Pd} = M$, and the dashed lines show the one standard deviation (0.39) locations. The uncertainty of magnitude determination from Pd is relatively small.

[16] The results in Figures 3a and 3b show that for earthquakes up to magnitude 7, there is clearly a determin-

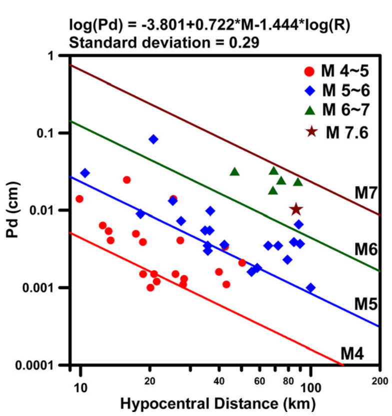

Figure 3a. Distribution of the observed Pd measurements. The four diagonal lines are calculated from the linear Pd-R relationship for magnitudes 4, 5, 6, and 7.

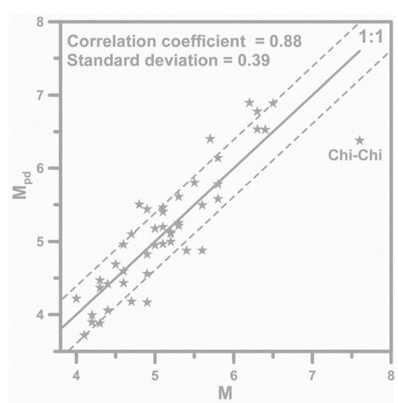

Figure 3b. Magnitudes determined from Pd versus the reported magnitudes M. Solid line shows the least squares fit and the two dashed lines show the range of one standard deviation.

istic relationship between the observed peak amplitudes in the first 3 seconds of the *P* waves and the magnitudes of earthquakes. However, it is rather intriguing that the magnitude 7.6 Chi-Chi earthquake violates this relationship and has a much lower magnitude of 6.38 predicted by *Pd*, as can be seen in Figure 3b. This is also significantly lower than the τ_c-estimated magnitude of 7.06. The magnitude of the Chi-Chi earthquake determined from *Pd* seems to imply a "cascade model" for the earthquake rupture process. However, this result remains unique in all the data we have analyzed, and more observations are needed.

4. Discussion and Conclusion

[17] In this study, we determined the relationships between earthquake magnitudes and the characteristic parameters τ_c and *Pd* observed from the first three seconds of the *P* waves. A good linear relationship is found between the characteristic periods τ_c of the initial *P* waves and the magnitudes of earthquakes. This linear relationship persists throughout the entire magnitude range in our study, showing a consistency with the earthquake initiation model proposed by *Olson and Allen* [2005] in which the final magnitude of an earthquake is to a large extent determined by the earliest stage of the rupture process. This result allows us to use the characteristic periods (or dominant frequency) observed from the first 3 seconds of the *P* waves at a single station to estimate the magnitudes of earthquakes. To avoid large uncertainly, previous studies [*Kanamori*, 2005; *Wu and Kanamori*, 2005a, 2005b] have suggested that a threshold level should be established when τ_c is used in EEW practice, since only large earthquakes can produce large τ_c.

[18] We have also found a good linear relationship among the logarithmic peak amplitude *Pd*, the magnitude, and the logarithmic hypocentral distance, for earthquakes up to at least magnitude 6.5. The effectiveness of the peak amplitude observation for large earthquakes ($M > 7$) remains to be examined. Although the *Pd*-estimated magnitude M_{Pd} may not predict the magnitudes of large earthquakes, it does correlate well with the catalog magnitude for $M < 6.5$ events with a small uncertainty of about 0.4. Since *Pd* is an amplitude parameter, most of the uncertainty could well come from the effect of the *P* wave radiation pattern. With a good grasp of the relationship between *Pd* and radiation pattern, *Pd* can be a very good instrument in magnitude determination for M < 6.5 events. Thus, we propose that a combination of *Pd* and τ_c analyses to be used for single station or onsite EEW operation.

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