GEOPHYSICAL RESEARCH LETTERS, VOL. 35, L20301, doi:10.1029/2008GL035611, 2008

A comparison of τ_c and τ_p^{max} for magnitude estimation in earthquake early warning

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Received 11 August 2008; revised 9 September 2008; accepted 15 September 2008; published 16 October 2008.

[1] We determined the τ_c and τ_p^{max} parameters from the K-NET strong motion records of 16 earthquakes in Japan with moment magnitude (M_w) ranging from 6.0 to 8.3. A 0.075 Hz high-pass Butterworth filter was applied for determination of τ_c based on our previous studies. It was found that different pole selections of the Butterworth filter lead to different uncertainty in magnitude determination. Our results show that using two poles in the filters results in the best magnitude estimates, i.e., minimized the standard deviation in magnitude determination in comparison to M_w using τ_c. The τ_p^{max} parameters (Allen and Kanamori, 2003) were also determined with the same dataset using the Wurman et al. (2007) procedure. It was found that τ_p^{max} values obtained from this dataset, and using the Wurman procedure, had a larger uncertainty. However, when a 0.075 Hz high-pass Butterworth filter with five poles was added, the uncertainty in τ_p^{max}-derived magnitude estimates decreased minimizing the standard deviation in magnitude determination using τ_p^{max}. This difference in the behavior of τ_c and τ_p^{max} can be used to further reduce the uncertainty in rapid magnitude determination for earthquake early warning. When the magnitude estimations from τ_c and τ_p^{max} of each event are averaged to provide a new magnitude estimate, the standard deviation in magnitude estimates is reduced further to 0.27 magnitude units. **Citation:** Shieh, J.-T., Y.-M. Wu, and R. M. Allen (2008), A comparison of τ_c and τ_p^{max} for magnitude estimation in earthquake early warning, *Geophys. Res. Lett.*, *35*, L20301, doi:10.1029/2008GL035611.

1. Introduction

[2] A central component of earthquake early-warning (EEW) systems is the determination of the magnitude and location of an earthquake as soon as possible and before destructive energy arrives. *Nakamura* [1988] first introduced the concept of using the frequency content of the initial few seconds of P-wave arrivals. He observed that larger events cause initial ground motion with longer periods than smaller events. Average ground motion period τ_c and dominant ground motion period τ_p^{max} are two important parameters frequently used to estimate the magnitude in EEW [e.g., *Allen and Kanamori*, 2003; *Kanamori*, 2005; *Olson and Allen*, 2005; *Wu and Kanamori*, 2005a, 2008a, 2008b; *Wu et al.*, 2007; *Wurman et al.*, 2007; *Olivieri et al.*, 2008]. One measure of P-wave frequency content is τ_c which uses the first 3 seconds of P-wave data. The results of *Wu and Kanamori* [2005a, 2005b, 2008a, 2008b] and *Wu et al.* [2006, 2007] show a good relationship between τ_c and M_w determined from data collected from Japan, Taiwan and southern California. This suggests that it is possible to estimate the magnitude 3 seconds after the P-wave arrival with the τ_c method.

[3] Building of the results of *Allen and Kanamori* [2003] in southern California, *Olson and Allen* [2005] also found a good scaling relationship between τ_p^{max} and M_w for a global earthquake dataset. While they used up to 4 sec of P-wave data, τ_p^{max} values for most of the records occurred within 2 seconds of the P-wave arrival. This relationship between τ_p^{max} and M_w also allows estimation of magnitude from the first few seconds of P-wave data. The fact that their observations were made prior to the termination of the earthquake rupture was also interpreted as suggesting that earthquake rupture is deterministic. This interpretation remains controversial. *Rydelek and Horiuchi* [2006] used a dataset of earthquakes with M > 6.0 from Japan to investigate the proposed scaling relation and argued that there was no obvious scaling relation between τ_p^{max} values and magnitude.

[4] Here we focus on the applicability of both the τ_c and τ_p^{max} parameters for EEW. We use a dataset that is similar to that of *Rydelek and Horiuchi* [2006], and compute τ_c and τ_p^{max} values from the vertical acceleration component of the K-NET strong motion records collected in Japan from 1997 to 2008. There are more than 1000 K-NET stations across Japan, and 16 events were selected in this study (Figure 1). We use the same dataset to determine both τ_c and τ_p^{max} and compare the performance of these parameters as magnitude estimators. We also experiment with the frequency band within which τ_c and τ_p^{max} are determined and find that this plays an important role in the robustness of magnitude estimates.

2. Data

[5] The purpose of EEW is to issue a warning before strong ground motion of a destructive earthquake comes. Thus, sixteen larger earthquakes with $M_w \geq 6$ (Table S1 in the auxiliary material) were chosen for analysis in this study.[1] The criteria for selecting events was: (1) events of $6 \leq M_w < 7$ with focal depth less than 30 km and at least six records within an epicentral distance of 70 km, and (2) events of $M_w \geq 7$ with focal depth less than 70 km and at least six records within an epicentral distance less than 200 km. Earthquakes with less than 6 records are not included in this analysis. In this study we use 3 seconds

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0094-8276/08/2008GL035611$05.00

[1]Auxiliary materials are available in the HTML. doi:10.1029/2008GL035611.

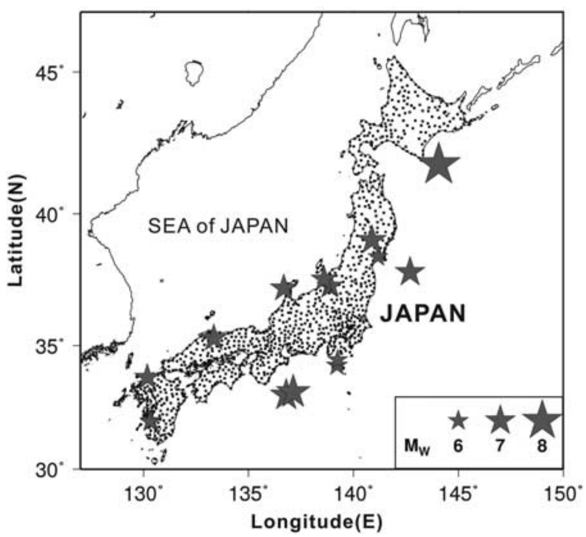

Figure 1. Epicenter distribution of events (grey stars) used in this study. Small squares show the locations of K-NET stations.

of data in our determination of τ_c and τ_p^{max}. Given that it is not possible to determine periods greater than ~12 seconds, i.e., our 3 second data window constitutes ¼ of the wavelength, we apply a 0.075 Hz high-pass filter, and also discard any observations greater than 10 seconds period. Considering the purpose of EEW in this study, for each event, we use the averaged value from the six waveform records with valid τ_c or τ_p^{max} nearest to the epicenter.

3. The τ_c Method

[6] τ_c is a measure of the average period of ground motion within some specified time window. It was first introduced by *Kanamori* [2005] and is a modified version of the method originally developed by *Nakamura* [1988]. The period parameter τ_c is calculated from the first several seconds of P-wave data as follows:

$$\tau_c = 2\pi / \sqrt{\left[\int_0^{t_0} \dot{u}^2(t)dt\right] \Big/ \left[\int_0^{t_0} u^2(t)dt\right]} \quad (1)$$

where u is the high-pass filtered displacement of the vertical component ground motion and $\dot{u}$ is the velocity differentiated from u. Following a series of studies [*Wu and Kananmori*, 2005a, 2005b, 2008a, 2008b; *Wu et al.*, 2006, 2007], the waveforms have a 0.075 Hz high-pass Butterworth filter applied to the velocity component during the procedure of τ_c determination (see Text S1, section S1a). A 3 seconds time window starting from first P-wave arrival is set to determine the τ_c in this study, i.e., τ_0 in equation (1) is set as 3 seconds after the P-wave arrival.

[7] In order to study the effect of different numbers of poles in the 0.075 Hz high-pass Butterworth filter, we tested filters with 1 through 6 poles (Figure S1 shows amplitude response curves) and examined the relationship between τ_c and M_W. We average the τ_c values from the six closest waveform records to each event and determined the linear relation between M_w and the averaged τ_c values using least-squares. Figure 2 shows the results of applying filters with 2 and 5 poles. Generally, the τ_c values with a small number of poles have larger slope versus M_W, which is good for magnitude estimation, but they also have a larger scatter (Figure 2a). A larger number of poles results in a smaller slope versus M_W, but with a smaller scatter (Figure 2b).

4. The τ_p^{max} Method

[8] τ_p^{max} was introduced by *Allen and Kanamori* [2003] (in which it is called T_p) and was applied to a global seismic

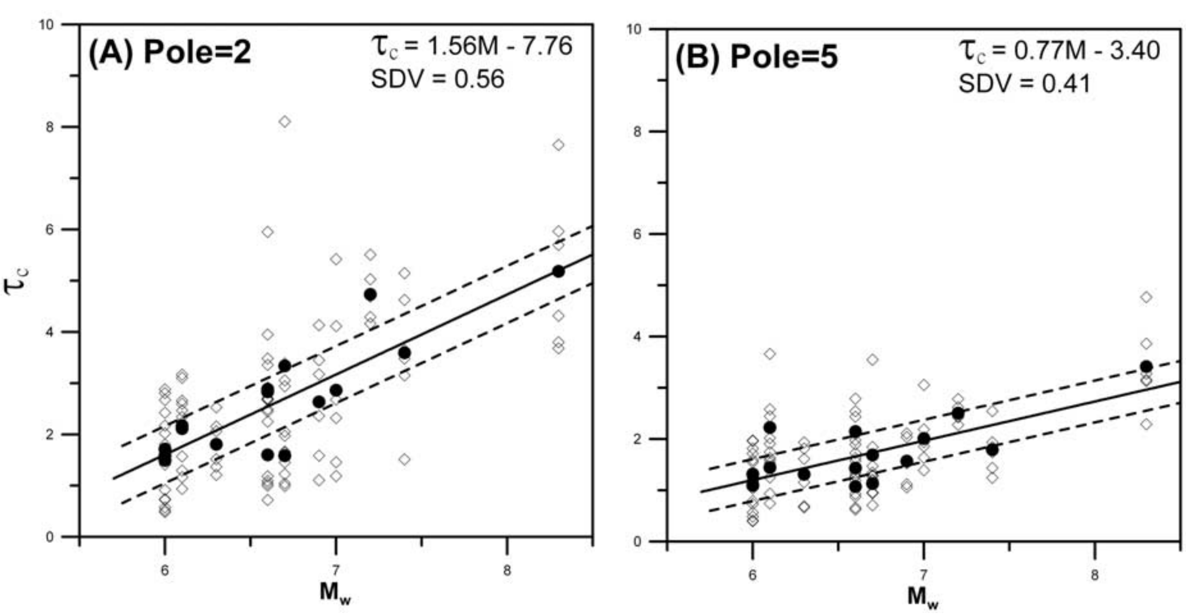

Figure 2. The τ_c estimated with (a) two poles and (b) five poles. A 0.075 Hz high-pass Butterworth filter was applied. Open diamonds represent the τ_c of each record, and solid circles represent the averaged τ_c values from records of the same events. Solid line shows the least-squares fit and the two dashed lines show the range of one standard deviation.

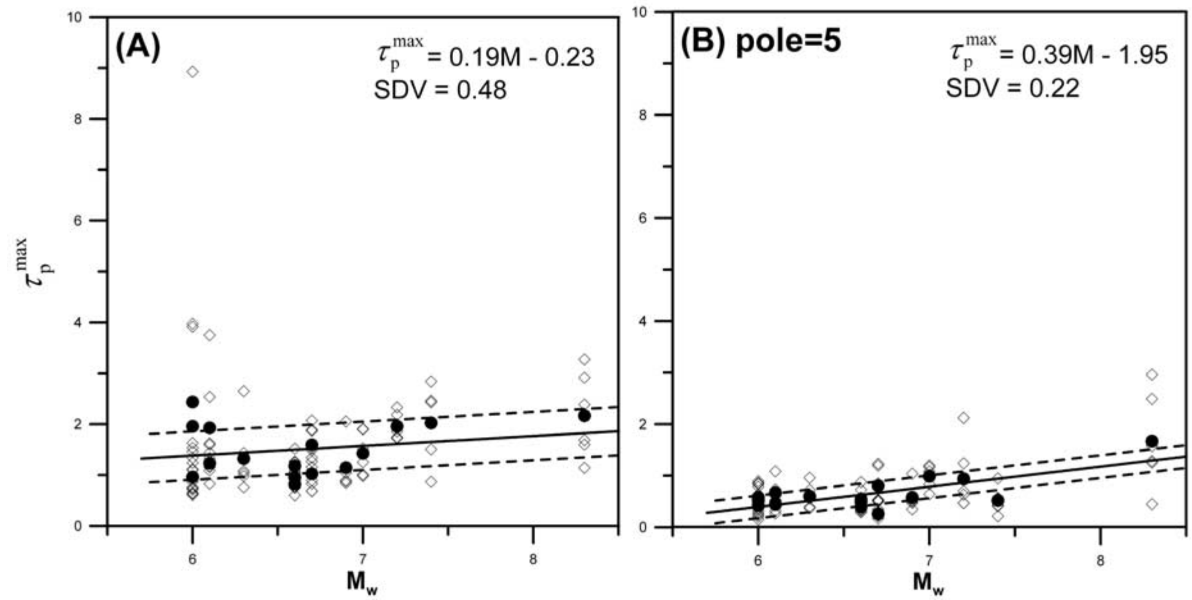

Figure 3. The τ_p^{max} estimated (a) by the original method of *Allen and Kanamori* [2003] and (b) by adding a five pole high-pass Butterworth filter at 0.075 Hz. Open diamonds represent the τ_p^{max} values of each record, and solid circles represent the average τ_p^{max} values from records of the same events. Solid line shows the least-squares fit and the two dashed

dataset by *Olson and Allen* [2005]. It is nearly identical to the original concept proposed by *Nakamura* [1988]. While the purpose of τ_p^{max} is the same as for τ_c in that it is a measure of frequency content, the approach is quite different. τ_c is determined by selecting a specific time window, 3 seconds in this case, and measuring the frequency content of the entire selected window. τ_p is a timeseries determined recursively and continuously from the seismic waveform. As such τ_p at any given time contains information about the frequency content of the entire waveform up to the given point in time, though the contribution of a given waveform segment decreases with time. This makes τ_p^{max} a dominant period parameter of ground motion while τ_c is an average period parameter. τ_p^{max} is also a ratio of the velocity and acceleration signals, while τ_c is the ratio of displacement and velocity signals.

[9] The parameter τ_p is computed by

$$\tau_i^p = 2\pi\sqrt{\frac{X_i}{D_i}}$$

where $X_i = \alpha X_{i-1} + x_i^2$

$$\text{and}\quad D_i = \alpha D_{i-1} + \left(\frac{dx}{dt}\right)_i^2 \qquad (2)$$

x_i is the velocity signal to which both high- and low-pass filters have been applied [*Wurman et al.*, 2007] (see Text S1, section S1b) and α is a smoothing constant which is set as 0.99 in this study. It is α that determines how quickly the contribution of a given segment of the time series to τ_p decreases with time. τ_p is computed at every time step and the maximum value, τ_p^{max}, within some time window is chosen to be the parameter used to estimate magnitude for EEW. In this study the time window used was 3 seconds for similarity with τ_c. τ_p^{max} is therefore the maximum value of τ_p within 3 seconds of the P-wave arrival. τ_p^{max} is selected from the time window starting at 0.05s rather than from 0.00s because of the recursive nature of the τ_p calculation as discussed by *Olson and Allen* [2005]. As with the τ_c vs. M_w relations in this study, the linear relation is shown by the least-squares fit between M_w and averaged values of τ_p^{max} from the same six records for each earthquake.

[10] Figure 3a shows τ_p^{max} values for the 16 earthquakes in this study. While τ_p^{max} increases with M_w, there is a large scatter in individual station observations for several of the smallest events resulting in the larger averaged τ_p^{max} values than for the larger events. This scatter is likely attributed to processing problems for smaller signal-to-noise ratio waveforms. Using the appropriate filter reduces the scatter. As with τ_c, we tried to apply a high-pass Butterworth filter at 0.075 Hz in the τ_p^{max} calculation. Figure 3b shows τ_p^{max} when five poles are used. This has the effect of narrowing the frequency band included in the τ_p^{max} calculation. The standard deviation of least-squares fitting of τ_p^{max} versus M_W decreases from 0.48 to 0.22 with the application of this filter.

5. Discussion and Conclusions

[11] The present study demonstrates that the filter application plays an important role in the calculation of τ_c and τ_p^{max}. In order to determine the best pole setting for the 0.075 Hz high-pass filter, relationships of τ_c and τ_p^{max} versus M_w were analyzed by least-squares fitting for pole values from 1 to 6. In the EEW application, we use the equation of least-squares fitting of τ_c or τ_p^{max} to estimate magnitude of an event. Since the purpose of these methods is to estimate the magnitude, standard deviations of estimated magnitude were used as the index to compare

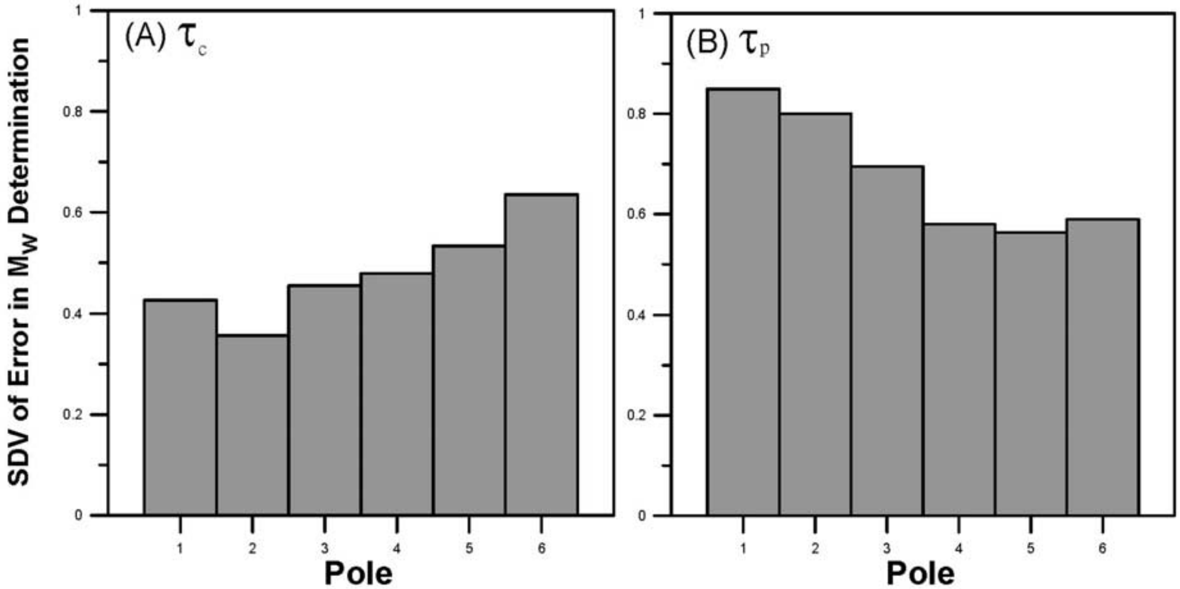

Figure 4. Standard deviations in magnitude estimation using τ_c and τ_p^{max} for different numbers of poles in the 0.075 Hz high-pass Butterworth filter.

results of different pole values. As shown in Figure 4 the best magnitude estimates are obtained from τ_c when the number of poles equals 2, which results in a standard deviation of 0.36 in the magnitude estimation. For τ_p^{max}, 5 poles had the best result in magnitude estimation resulting in a standard deviation of 0.56. Without the 0.075 Hz high-pass filter the standard deviation in the magnitude estimate from τ_p^{max} is 2.48.

[12] Based on this result, τ_c approach seems more robust than τ_p^{max}. However, these two parameters are based on the same concept from *Nakamura* [1988]. For the τ_c calculation, a three seconds window after P arrival is used, while the τ_p calculation is recursive. Thus, the τ_p value may be influenced by signals before P arrival. To abate this influence, we calculated τ_p by setting waveform values to zero prior to 0.05 seconds after the P-wave arrival. τ_p^{max} was then determined from the τ_p timeseries up to 3 seconds after P-wave arrival. A 0.075Hz high-pass Butterworth filter with 5 poles was also applied. Figure 5 shows the τ_p^{max} results. The uncertainty in magnitude estimation is decreased resulting in a standard deviation in the magnitude estimate of 0.40. This uncertainty is essentially the same as the uncertainty from the τ_c method.

[13] These tests have shown the importance of filter application in the calculation of τ_c and τ_p^{max}. We find that adding a 0.075 Hz high-pass Butterworth filter with a sharp cutoff in frequency (5 poles) is optimal for τ_p^{max} analysis enhancing the relationship between τ_p^{max} and magnitude. For τ_c, 2 poles have a best result in magnitude estimation. The different filter applications to τ_p^{max} analysis results in a diversity of measurements that may be the cause of the controversy introduced by *Rydelek and Horiuchi* [2006]. While there is difference in the behavior of τ_c and τ_p^{max}, when the appropriate specific procedure is applied, both methods have good linear trends with M_w. This suggests that it may be useful to include both τ_c and τ_p^{max} in the estimation of magnitude in earthquake early warning systems. The magnitude estimates of τ_c with two poles and τ_p^{max} with five pole calculated from 0.05 seconds after P arrival could be averaged to provide a more robust magnitude estimate. This average magnitude estimation has a lower uncertainty than either τ_c or τ_p^{max} alone. The

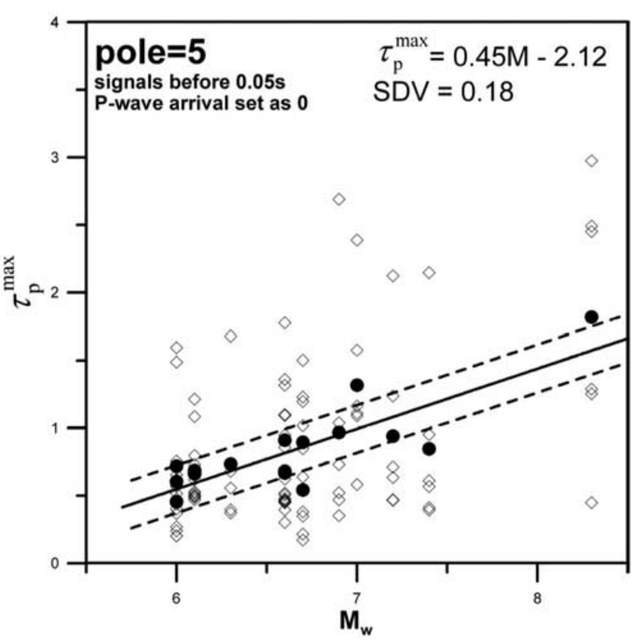

Figure 5. The τ_p^{max} estimated in the same way as Figure 3b, i.e., applying a five pole high-pass Butterworth filter at 0.075 Hz, but with the signals before 0.05s after the P-wave arrival set to zero. Hollow diamonds represent the τ_p^{max} values of each record, and solid circles represent the average τ_p^{max} values from records of the same events. Solid line shows the least-squares fit and the two dashed lines show the range of one standard deviation.

standard deviation of this average magnitude estimate is 0.27 magnitude units.

[14] **Acknowledgments.** We would like to thank M. Olivieri and one anonymous reviewer for their valuable comments. This research was supported by the Central Weather Bureau and the National Science Council of the Republic of China (NSC96-2625-Z-002-025 and NSC95-2119-M-002-043-MY3). Support was also provided by the USGS NEHRP program (06HQAG0147). Figure 1 was made using Generic Mapping Tool [*Wessel and Smith*, 1991].

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