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Magnitude determination using strong ground-motion attenuation in earthquake early warning

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[1] We use 1661 strong-motion accelerograms with peak ground acceleration (PGA) larger than 80 Gal (1 Gal = 1.0 cm/s^2) from 77 earthquakes recorded by the Taiwan Strong Motion Instrumentation Program (TSMIP) stations to derive a strong-motion attenuation relationship. This relationship can be used to dynamically define a "M_{pga} magnitude" for earthquakes using earthquake locations determined by earthquake early warning process. The M_{pga} magnitude using this strong-motion attenuation relationship corresponds well with M_w given a sufficient number of PGA readings. MEMS (Micro Electro Mechanical Systems) acceleration sensor could be widely used for ground motion monitoring purposes. Thus, we propose that once a large earthquake has begun, that we might be able to use strong, near-field (tens of kilometers) PGA values to quickly estimate the earthquake's magnitude, which would improve earthquake early warning. **Citation:** Lin, T.-L., and Y.-M. Wu (2010), Magnitude determination using strong ground-motion attenuation in earthquake early warning, *Geophys. Res. Lett.*, *37*, L07304, doi:10.1029/2010GL042502.

1. Introduction

[2] When a large earthquake occurs, an earthquake early warning (EEW) system can alert populations, sensitive facilities such as nuclear reactors, gas pipelines, and public transportation systems, ahead of the arrival of strong ground shaking. The idea of an EEW system was proposed more than one hundred years ago by *Cooper* [1868] for San Francisco, California. An early warning leading time can be a few seconds to a few tens of seconds depending on the distance between the earthquake and the target warning areas. Although this warning time window might seem short, it can be critical, as even a few seconds are sufficient to initiate pre-programmed EEW emergency safety responses. Therefore, EEW is a practical, effective approach to seismic risk mitigation on a short time-scale [*Kanamori et al.*, 1997; *Teng et al.*, 1997; *Wu and Teng*, 2002; *Allen and Kanamori*, 2003; *Kanamori*, 2005] as compared to the longer time-scale earthquake prediction.

[3] The two most commonly used types of EEW are regional and onsite warning systems. In regional EEW, the ground shaking characteristics recorded by seismic sensors closest to the earthquake rupture are used to predict strong ground motions at more distant target areas. In onsite EEW, the initial *P*-wave motion at a given site is used to predict the ground motions of the later *S* and surface waves (which commonly have higher amplitudes or destructive energy than that of the initial *P*-wave motion) at the same site or region where the onsite warning instruments are operating.

[4] An onsite EEW system must predict the earthquake's magnitude, and the ensuing peak ground motion rapidly and reliably. An average period parameter (τ_c) from the initial 3 seconds of the *P*-wave [*Kanamori*, 2005; *Wu and Kanamori*, 2005a; *Wu et al.*, 2007a], originally proposed by *Nakamura* [1988], and *Allen and Kanamori* [2003], can be used to predict the size of an earthquake. *Wu and Kanamori* [2008a, 2008b] showed that earthquake magnitude (M_w) could be estimated from τ_c for strong motion data from the Japan, Taiwan, and southern California records. *Wu and Kanamori* [2005b] and *Wu et al.* [2007a] showed that the peak initial displacement amplitude, Pd, from the first 3 seconds of the *P*-wave correlates well with the peak ground-motion velocity, PGV, observed at the same site, using strong motion data from Taiwan and southern California, respectively. Combining τ_c and Pd (τ_c × Pd), *Wu and Kanamori* [2005b] demonstrated that τ_c × Pd provides a more reliable indicator for indentifying damaging earthquakes.

[5] For regional EEW, rapid and reliable determination of earthquake magnitude is more difficult than the estimation of other parameters (such as earthquake location) because the shear wave portion may not completely arrive in a few to ten seconds time window [*Wu and Zhao*, 2006]. Using the real-time strong-motion network in Taiwan, *Wu et al.* [1998] proposed the M_{L10} method based on the first 10 seconds of the *P*-wave to determine earthquake magnitude that is well correlated to the local magnitude M_L for regional EEW purpose. In the regional EEW approach, *Wu and Zhao* [2006] showed, for earthquakes in southern California, that Pd is also a robust measurement for estimating the earthquake magnitude. Onsite EEW systems under development in Taiwan and southern California will use the τ_c method for magnitude determination.

[6] This study provides a novel, experimental concept to rapidly determine earthquake magnitude using strong ground-motion attenuation once an earthquake location is determined by the onsite [*Odaka et al.*, 2003] and regional [*Rydelek and Pujol*, 2004; *Horiuchi et al.*, 2005; *Cua and Heaton*, 2007; *Wurman et al.*, 2007] EEW techniques. We first define a strong ground-motion attenuation relationship for Taiwan for large crustal earthquakes with PGA observations larger than 80 Gal. Then, we use the resulting strong motion attenuation relationship to invert for the M_{pga} magnitude. Our estimated M_{pga} magnitudes agree well with M_w for large earthquakes with sufficient numbers of PGA observations. We propose that the magnitude determination method presented in this study might be integrated into current operating EEW rapid reporting systems to provide

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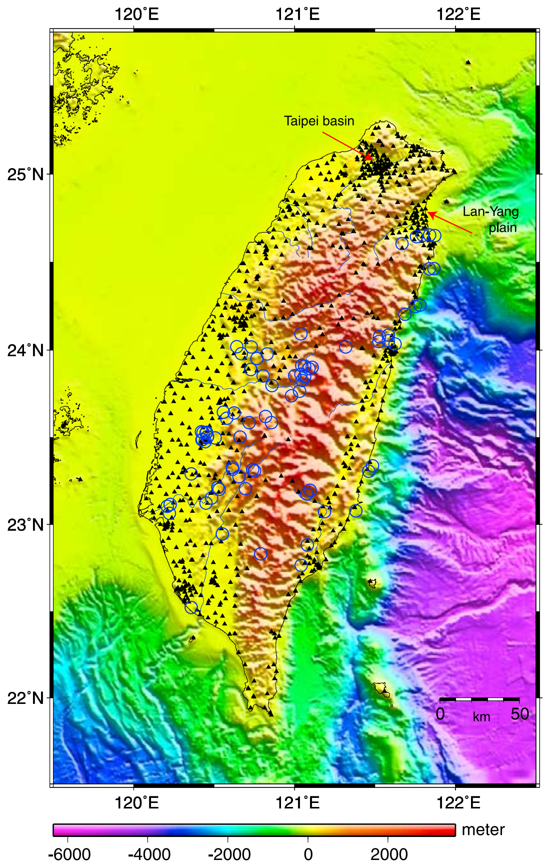

Figure 1. Locations of seismic stations (triangles) of the TSMIP seismic network. The epicenter of 77 earthquake events used in this study are show as the open circles. The TSMIP stations are limited in the high-relief mountain ranges.

an additional EEW magnitude parameter, thus building more redundancy in the EEW system.

2. Strong Ground-Motion Attenuation

[7] Taiwan, located in the western part of the Pacific Rim seismic belt, is situated in the collision boundary zone between the Philippine Sea and Eurasian continental plates. Therefore, the seismicity in Taiwan is considerably high. Considering its high population density and seismic activity, Taiwan is an area that is vulnerable to serious seismic hazard. A large magnitude earthquake such as the 1999 Chi-Chi event could once again strike Taiwan and cause catastrophic loss of life and massive economic damage.

[8] The TSMIP [*Liu et al.*, 1999] operated by the Taiwan Central Weather Bureau (CWB), consists of over 800 free-field seismic stations densely distributed throughout Taiwan as of 2008 (Figure 1). The TSMIP has an average station spacing of about 5 km throughout most populated areas, except that they are much less densely distributed in the higher relief mountain ranges. Each TSMIP station is equipped with a three-component, force-balance accelerometer with a 16- or 24-bits resolution digitizer, with a sampling rate of 200 Hz or higher. This instrument's acceleration response is flat from DC to about 50 Hz.

[9] For this study, we used shallow crustal earthquakes with PGA values (on ay least one of the three components) exceeding 80 Gal, recorded by the TSMIP stations from 1993 to 2008; since either EEW or rapid reporting is principally and practically applied to large damaging earthquakes. A PGA value larger than 80-Gal corresponds to a CWB intensity scale of V (80–250 Gal) in Taiwan [*Wu et al.*, 2003], or to a Modified Mercalli intensity scale [*Wald et al.*, 1999] value of VI (92–180 Gal). Large, shallow inland earthquakes often cause the most serious damage. We assume that 42 km is the average depth to the Moho interface in Taiwan [*Tomfohrde and Nowack*, 2000; *Wu et al.* 2007b], and define a shallow earthquake as one having a focal depth of less than 42 km. In fact, most inland or offshore (distance to shoreline < 5 km) earthquakes that cause ground shaking to a maximum PGA > 80 Gal have focal depths shallower than 25 km in Taiwan. We ended up with 1661 PGA readings larger than 80 Gal recorded by TSMIP stations, from 77 crustal earthquakes (Figure 1 and Table S1) to use to derive the strong ground-motion attenuation relationship for this study. We then used these observations to invert for the constants in our assumed "M_{pga} magnitude" parameterization.

[10] We chose a strong ground-motion attenuation relationship of the from:

$$\log_{10} PGA = a \log_{10}(r) + bM + c, \tag{1}$$

where r is the epicenter distance in km, M is the moment magnitude (M_w), a and b are empirical coefficients for geometrical spreading and magnitude, respectively, and c is a constant. Coefficients a, b, and c are the parameters to be determined from a regression analysis. The use of M_w instead of local magnitude (M_L) avoids the magnitude saturation of large earthquakes, which is particularly important as EEW and rapid reporting are mostly directed toward larger earthquakes. We have adopted the M_w that were reported by Harvard centroid moment tensor (CMT) project. For earthquakes without Harvard CMT solutions, mostly smaller than M_w 5.3, we have used the empirical equation (equation (2)) to convert M_L to M_w [*Chen and Tsai*, 2008]:

$$M_L = -0.24 + 1.07 M_W \pm 0.31. \tag{2}$$

As these earthquakes are too small to saturate M_L, we assume that this formula provides a good estimate of M_w for this subset of low magnitude events. The epicenter distance (r) is used because most large damaging earthquakes in Taiwan are located in the shallow crust and in the current processes of EEW, the estimation of epicenter location might be more practical and accurate than real time estimates of hypocentral location.

[11] Equation (1) can be rewritten in matrix form as:

$$\begin{bmatrix} \log_{10} r_1 & M_1 & 1 \\ \log_{10} r_2 & M_1 & 1 \\ \vdots & \vdots & \vdots \\ \log_{10} r_{1661} & M_{17} & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \log_{10} PGA_1 \\ \log_{10} PGA_2 \\ \vdots \\ \log_{10} PGA_{1661} \end{bmatrix} \tag{3}$$

or Gm = d. Equation (3) presents a typical overdetermined inversion problem. The vector of unknowns (m) were found

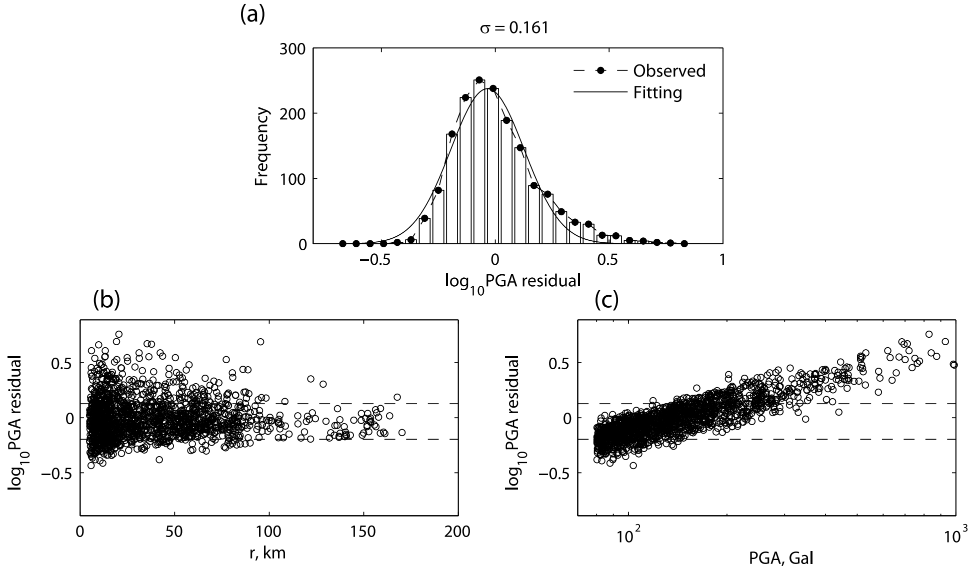

Figure 2. (a) Distribution of the residuals between observed and predicted (equation (4)) accelerations. The variance of the residual distribution was used for the error estimation. Residuals are plotted against (b) epicenter distance and (c) logarithms of observed PGA.

through generalized inverse matrix of G (G^{-g}) using singular value decomposition (SVD) [*Menke*, 1984; *Miao and Langston*, 2007]. Unlike most common attenuation relationships, anelastic attenuation (linear *r* term) is not considered in the right-hand side of equation (1). This is because we found that variances (standard deviations) of the model parameters (*a* and *c*) significantly increase in the SVD computations if we include this term. Moreover, in studies of M_L determination [*Wu et al.*, 2005] and Pd attenuation [*Wu and Zhao*, 2006], both found that the linear *r* term is not statistically significant. Using 1661 PGA readings (> 80 Gal) from 77 events recorded by the dense TSMIP stations in Taiwan, we thus obtain a strong ground-motion attenuation relationship of:

$$\log_{10} PGA = -0.395 \log_{10}(r) + 0.125M + 1.979 \pm 0.161. \quad (4)$$

The standard deviation (0.161) in equation (4) is one standard deviation of the normal distribution that best fits the frequency of $\log_{10}PGA$ residuals (Figure 2). Note that for the PGA readings observed at epicenter distances less than 3 km (i.e., high PGA) are not used in the SVD inversion since these PGA readings usually have abnormal residuals. Figures 2b and 2c indicate that in general the PGA residuals increase as the epicenter distances and the values of $\log_{10}PGA$ increase.

3. Magnitude Determination

[12] Given an epicenter location produced by either a regional or an onsite EEW system immediately after the occurrence of a large earthquake; the M_{pga} magnitude can be simultaneously and dynamically estimated by solving for *M* in equation (4) for each PGA observation as it becomes available. Figure 3 gives four examples of the M_{pga} magnitude history in real time given by equation (4), with increasing numbers of PGA observations, compared to the reported M_w magnitude for the event. Figure 3a is an example for the 1999 Chi-Chi, Taiwan earthquake (M_w = 7.6) which is to date the largest inland recorded event, responsible for the largest number of death, and damage in Taiwan's natural hazard history. During the Chi-Chi earthquake main shock, a total of 227 acceleration records having PGA values larger than 80 Gal were registered by the TSMIP stations. We consider all four example earthquakes to be large enough to test our implementation of an EEW and rapid reporting system.

[13] The M_{pga} magnitude, resulting from our inversion of the strong-motion attenuation relationship (equation (4)) generally corresponds well with the reported M_w magnitude (Figure 3). As the number of PGA recordings increases for a given event, its M_{pga} magnitude approaches the reported M_w magnitude for that earthquake. In general, about 20–30 PGA recordings provide a stable estimate of M_w, suitable for reporting as part of an EEW system. The distribution maps of PGA recording (Figure 3) show that given the current TSMIP station density, an area enclosed by a circle of a radius of about 50 km will provide a satisfactory number of PGA readings to produce a stable M_{pga}. Even though PGA values smaller than 80 Gal were not used to obtain equation (4), the M_{pga} magnitudes agree well with the reported M_w magnitudes.

4. Discussion

[14] EEW and rapid reporting systems are applied to earthquakes large enough to cause disastrous damages. Therefore, it is desirable to have rapid earthquake magnitude determinations that are accurate for the very large earthquakes (says $M_w > 6.5$). Figures 4a and 4b compare M_{pga} magnitudes with M_w magnitudes for earthquakes larger than M_w = 4.5, and for those larger than M_w = 6.0, respectively. Figure 4 indicates that the M_{pga} magnitudes estimated from the PGA values for the earthquakes of $M_w > 6.0$ have the smaller standard deviations (SD) than those for the earthquakes of $M_w > 4.5$. We attribute this result to the larger

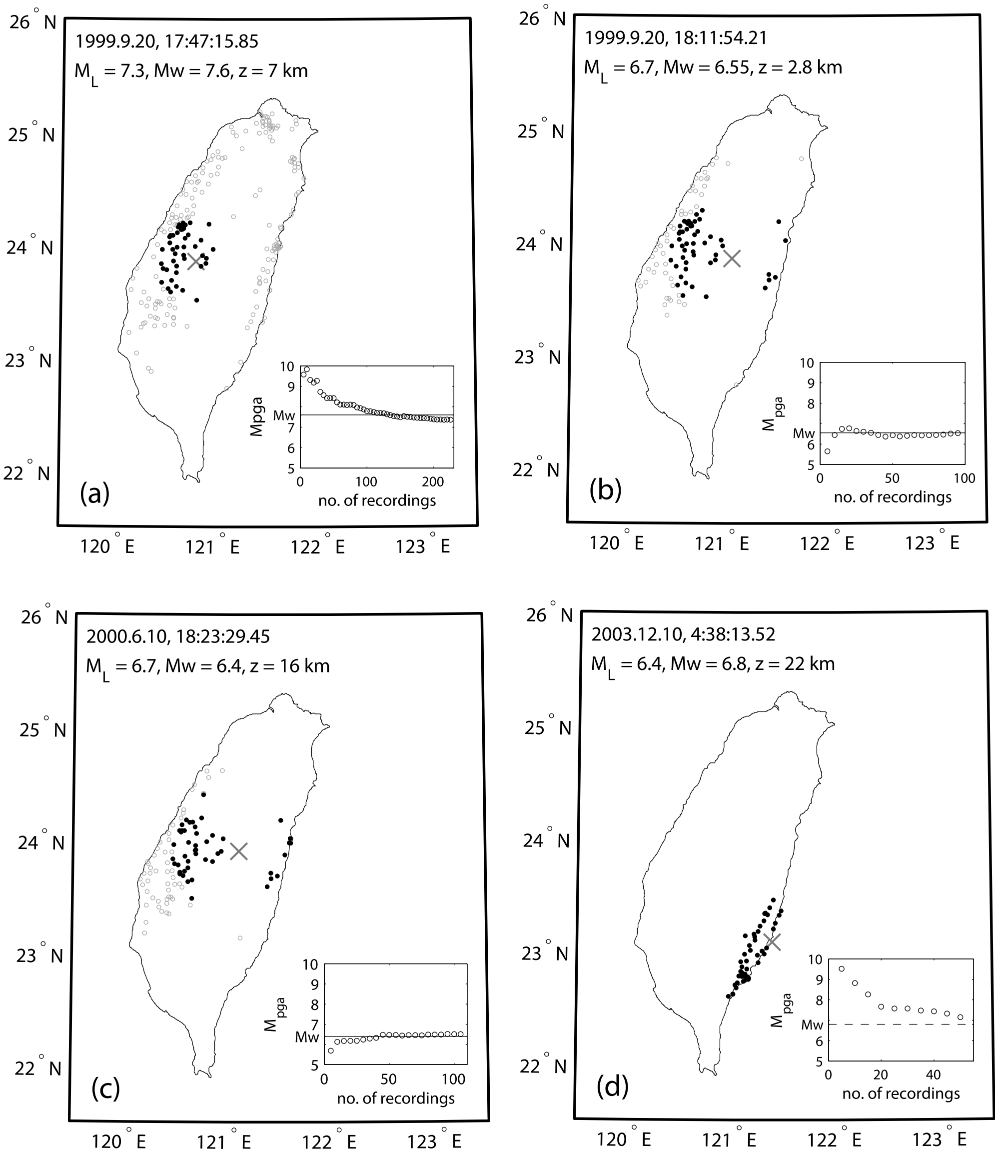

Figure 3. (a–d) The maps of the TSMIP stations recording the PGA values larger than 80 Gal and the relation between the M_{pga} magnitude and the numbers of the PGA recording for the four large earthquakes. The crosses and circles in the maps indicate the epicenters and the TSMIP station locations with the PGA values larger than 80 Gal. The close circles indicate the closest 50 stations to the epicenters.

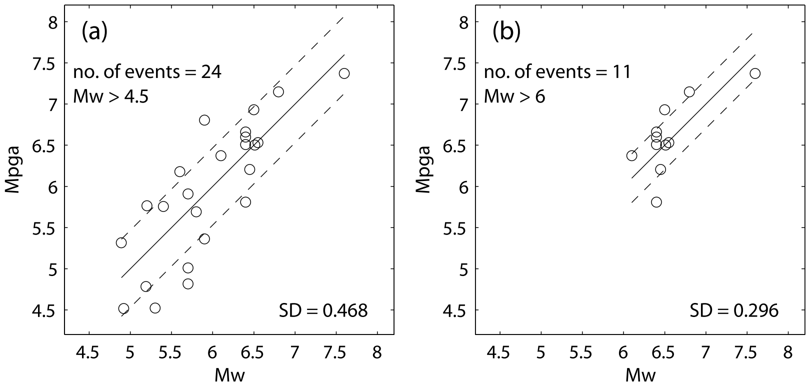

Figure 4. Comparisons of the M_{pga} magnitude to M_w magnitude for earthquakes larger than (a) $M_w = 4.5$ and (b) $M_w = 6.0$, respectively.

number of PGA recordings and to the better epicenter distance coverage for the larger magnitude earthquakes. Note that in Figure 4 the number of the PGA recordings of each earthquake is required to be greater than 20 recordings. Due to the lack of events with $M_w > 7.0$, it is difficult to identify the saturation problem [*Schmedes and Archuleta*, 2008] of the M_{pga} magnitude in this study.

[15] Figures 3 and 4 imply that more seismic stations (i.e., recordings) within a few tens of kilometers of an earthquake's epicenter will improve the accuracy of the M_{pga} estimate of M_w. Therefore, a dense seismic network is critical for the effective use of our proposed magnitude determination method in real time, on-line EEW practice. The MEMS acceleration sensors that have been recently introduced into seismic applications [*Holland*, 2003] are miniature, low cost, and ideal for recording near-field, high-frequency ground motions. The magnitude determination method proposed in this paper would be extremely fast and accurate if based on data from dense seismic networks via extensive installation of MEMS accelerometers. MEMS accelerometer networks have been successfully tested by the Quake-Catcher Network, which may have the potential for EEW [*Cochran et al.*, 2009]. In addition, more robust determination of earthquake magnitude is expected by combining the regional-oriented M_{pga} method and the M_{Pd} method introduced by *Wu and Zhao* [2006], with that the onsite-determined magnitude using τ_c to have more redundancy of magnitude determination in future EEW systems.

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References

Allen, R. M., and H. Kanamori (2003), The potential for earthquake early warning in southern California, *Science*, *300*, 786–789, doi:10.1126/science.1080912.

Chen, K.-P., and T.-B. Tsai (2008), A catalog of Taiwan earthquakes (1900–2006) with homogenized M_w magnitudes, *Bull. Seismol. Soc. Am.*, *98*, 483–489, doi:10.1785/0120070136.

Cochran, E. S., J. F. Lawrence, C. Christensen, and R. S. Jakka (2009), The Quake-Catcher Network: Citizen science expanding seismic horizons, *Seismol. Res. Lett.*, *80*, 26–30, doi:10.1785/gssrl.80.1.26.

Cooper, J. D. (1868), Letter to editor, *San Francisco Daily Evening Bulletin*, 3 Nov.

Cua, G., and T. Heaton (2007), The Virtual Seismologist (VS) method: A Bayesian approach to earthquake early warning, in *Earthquake Early Warning Systems*, edited by P. Gasparini, G. Manfredi, and J. Zschau, pp. 85–132, Springer, Berlin.

Holland, A. (2003), Earthquake data recorded by the MEMS accelerometer, *Seismol. Res. Lett.*, *74*, 20–26.

Horiuchi, S., H. Negishi, K. Abe, A. Kamimura, and Y. Fujinawa (2005), An automatic processing system for broadcasting earthquake alarms, *Bull. Seismol. Soc. Am.*, *95*, 708–718, doi:10.1785/0120030133.

Kanamori, H. (2005), Real-time seismology and earthquake damage mitigation, *Annu. Rev. Earth Planet. Sci.*, *33*, 195–214, doi:10.1146/annurev.earth.33.092203.122626.

Kanamori, H., E. Hauksson, and T. Heaton (1997), Real-time seismology and earthquake hazard mitigation, *Nature*, *390*, 461–464, doi:10.1038/37280.

Liu, K. S., T. C. Shin, and Y. B. Tsai (1999), A free-field strong motion network in Taiwan: TSMIP, *Terr. Atmos. Oceanic Sci*, *10*, 377–396.

Menke, W. (1984), *Geophysical Data Analysis: Discrete Inverse Theory*, Academic, Orlando, Fla.

Miao, Q., and C. A. Langston (2007), Empirical distance attenuation and the local magnitude scale for the central U.S, *Bull. Seismol. Soc. Am.*, *97*, 2137–2151, doi:10.1785/0120060188.

Nakamura, Y. (1988), On the urgent earthquake detection and alarm system (UrEDAS), in *Proceedings of the Ninth World Conference on Earthquake Engineering*, vol. 8, pp. 673–678, Jpn. Assoc. for Earthquake Disaster Prev., Tokyo.

Odaka, T., K. Ashiya, S. Tsukada, S. Sato, K. Ohtake, and D. Nozaka (2003), A new method of quickly estimating epicentral distance and magnitude from a single seismic record, *Bull. Seismol. Soc. Am.*, *93*, 526–532, doi:10.1785/0120020008.

Rydelek, P., and J. Pujol (2004), Real-time seismic warning with a two-station subarray, *Bull. Seismol. Soc. Am.*, *94*, 1546–1550, doi:10.1785/012003197.

Schmedes, J., and R. J. Archuleta (2008), Near-source ground motion along strike-slip faults: Insights into magnitude saturation of PGV and PGA, *Bull. Seismol. Soc. Am.*, *98*, 2278–2290, doi:10.1785/0120070209.

Teng, T. L., Y. M. Wu, T. C. Shin, Y. B. Tsai, and W. H. K. Lee (1997), One minute after: Strong-motion map, effective epicenter, and effective magnitude, *Bull. Seismol. Soc. Am.*, *87*, 1209–1219.

Tomfohrde, D. A., and R. L. Nowack (2000), Crustal structure beneath Taiwan using frequency-band inversion of receiver function waveforms, *Pure Appl. Geophys.*, *157*, 737–764, doi:10.1007/PL00001116.

Wald, D. J., V. Quitoriano, T. H. Heaton, and H. Kanamori (1999), Relationships between peak ground acceleration, peak ground velocity, and Modified Mercalli Intensity in California, *Earthquake Spectra*, *15*, 557–564, doi:10.1193/1.1586058.

Wessel, P., and W. H. F. Smith (1998), New, improved version of Generic Mapping Tools released, *EOS, Trans. Amer. Gophys. U.*, *79*, 579.

Wu, Y. M., and H. Kanamori (2005a), Experiment on an onsite early warning method for the Taiwan early warning system, *Bull. Seismol. Soc. Am.*, *95*, 347–353, doi:10.1785/0120040097.

Wu, Y. M., and H. Kanamori (2005b), Rapid assessment of damaging potential of earthquakes in Taiwan from the beginning of P waves, *Bull. Seismol. Soc. Am.*, *95*, 1181–1185, doi:10.1785/0120040193.

Wu, Y. M., and H. Kanamori (2008a), Development of an earthquake early warning system using real-time strong motion signals, *Sensors*, *8*(1), 1–9, doi:10.3390/s8010001.

Wu, Y. M., and H. Kanamori (2008b), Exploring the feasibility of on-site earthquake early warning using close-in records of the 2007 Noto Hanto earthquake, *Earth Planets Space*, *60*, 155–160.

Wu, Y. M., and T. L. Teng (2002), A virtual sub-network approach to earthquake early warning, *Bull. Seismol. Soc. Am.*, *92*, 2008–2018, doi:10.1785/0120010217.

Wu, Y. M., and L. Zhao (2006), Magnitude estimation using the first three seconds P-wave amplitude in earthquake early warning, *Geophys. Res. Lett.*, *33*, L16312, doi:10.1029/2006GL026871.

Wu, Y. M., T. C. Shin, and Y. B. Tsai (1998), Quick and reliable determination of magnitude for seismic early warning, *Bull. Seismol. Soc. Am.*, *88*, 1254–1259.

Wu, Y. M., T. L. Teng, T. C. Shin, and N. C. Hsiao (2003), Relationship between peak ground acceleration, peak ground velocity, and intensity in Taiwan, *Bull. Seismol. Soc. Am.*, *93*, 386–396, doi:10.1785/0120020097.

Wu, Y. M., R. M. Allen, and C. F. Wu (2005), Revised M_L determination for crustal earthquakes in Taiwan, *Bull. Seismol. Soc. Am.*, *95*, 2517–2524, doi:10.1785/0120050043.

Wu, Y. M., H. Kanamori, R. Allen, and E. Hauksson (2007a), Determination of earthquake early warning parameters, τ_c and P_d, for southern California, *Geophys. J. Int.*, *170*, 711–717, doi:10.1111/j.1365-246X.2007.03430.x.

Wu, Y. M., C. H. Chang, L. Zhao, J. B. H. Shyu, Y. G. Chen, K. Sieh, and J. P. Avouac (2007b), Seismic tomography of Taiwan: Improved constraints from a dense network of strong-motion stations, *J. Geophys. Res.*, *112*, B08312, doi:10.1029/2007JB004983.

Wurman, G., R. M. Allen, and P. Lombard (2007), Toward earthquake early warning in northern California, *J. Geophys. Res.*, *112*, B08311, doi:10.1029/2006JB004830.

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