Magnitude estimation using initial P-wave amplitude and its spatial distribution in earthquake early warning in Taiwan

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[1] We collected the strong-motion accelerograms from thirteen crustal earthquakes in Taiwan recorded by the Taiwan Strong Motion Instrumentation Program (TSMIP) stations to find the empirical relationship between the areas of high initial *P*-wave displacement and the corresponding earthquake magnitudes. We found that the logarithm of the area has a linear relation to the corresponding earthquake magnitude in M_w for study earthquakes with magnitudes between 5.4 and 6.5. We propose that this relationship might be able to rapidly define earthquake magnitude without knowing the earthquake location in regions with sufficient seismic station coverage and might have practical application in earthquake early warning (EEW) and rapid reporting systems. **Citation:** Lin, T.-L., Y.-M. Wu, and D.-Y. Chen (2011), Magnitude estimation using initial P-wave amplitude and its spatial distribution in earthquake early warning in Taiwan, *Geophys. Res. Lett.*, *38*, L09303, doi:10.1029/2011GL047461.

1. Introduction

[2] Taiwan, located in the western part of the Pacific Rim seismic belt, is situated in the collision boundary zone between the Philippine Sea and Eurasian continental plates. As a consequence, the seismicity in Taiwan is considerably high. A large magnitude earthquake such as the 1999 Chi-Chi event could once again strike Taiwan and cause catastrophic loss of life and massive economic damage. Over decades of development in EEW, studies have shown that EEW is a potential approach for real-time seismic risk mitigation [*Kanamori et al.*, 1997; *Teng et al.*, 1997; *Wu and Teng*, 2002; *Allen and Kanamori*, 2003; *Kanamori*, 2005]. Taiwan has been developing rapid reporting and EEW systems since a real-time strong-motion network was installed in 1995 by the Taiwan Central Weather Bureau (CWB) [*Wu et al.*, 1997].

[3] The common EEW approach to find earthquake location is based on the traditional locating methods. The on-line, real-time EEW location estimation merely uses fewer stations (arrivals), which are selected near the source location in order to speed up the collecting process without losing too much accuracy. On the other hand, earthquake magnitude is assumed to be correlated with the initial portion of the seismograms, mostly *P*-wave. Limited by the incomplete arrivals of the *S*-wave signals, therefore, the magnitude estimation inevitably possesses lower accuracy compared to the location estimation.

[4] The method of magnitude estimation in the present EEW operation in Taiwan is based on the M_{L10} [*Wu et al.*, 1998] approach. This approach involves calculating a local magnitude (M_{L10}) from data available within 10 seconds of the initial *P*-wave detection at stations in the vicinity of the earthquake source region [*Wu and Teng*, 2002], and using an empirical relationship between M_{L10} and traditionally calculated (non-real-time) M_L. This method has provided satisfactory magnitude estimations since its implementation in 2001 with a standard deviation of 0.28 magnitude unit [*Hsiao et al.*, 2009]. However, the reporting time of M_{L10} method leads to a "blind zone" with a radius of 70 km centered in the epicenter, in which warnings cannot be issued in a timely manner [*Hsiao et al.*, 2009]. The average reporting time of M_{L10} method is of about 20 seconds [*Hsiao et al.*, 2009]. The EEW information is not publicly distributed yet in the present EEW by CWB in Taiwan.

[5] In addition to the operational EEW system based on M_{L10}, CWB is also testing a second prototype EEW approach using the peak initial-displacement amplitude (P*d*) of the three-second window after the first *P*-wave arrival on the high-pass (0.075 Hz) filtered vertical displacement seismogram [*Hsiao et al.*, 2011]. Using the P*d* attenuation relationship with hypocentral distance, *Wu and Zhao* [2006], and *Hsiao et al.* [2011] showed that P*d* is also a useful parameter in estimating earthquake magnitude (M_{Pd}) for earthquakes in southern California and Taiwan, respectively. One prerequisite during the M_{Pd} process is the earthquake location for calculating the hypocentral distance. Consequently, any errors and/or uncertainties of the event location will contribute towards errors and/or uncertainties in magnitude estimation.

[6] Using the accelerograms recorded by the TSMIP stations, *Lin and Wu* [2010] found that the logarithms of the areas inside a variety of PGA (peak ground acceleration) contours ranging from 100 to 400 Gal (1 Gal = 1.0 cm/s^2) have a linear relation to the corresponding earthquake magnitudes. They proposed that this area-magnitude relationship [*Teng et al.*, 1997] could be used to rapidly estimate earthquake magnitude without knowing the earthquake location in regions with an adequate seismic station density. However, since PGA is used in this magnitude estimation, the PGA collecting time is lengthened by considering the complete source rupture, especially for a larger magnitude earthquake. Unlike the P*d*-attenuation method [*Wu and Zhao*, 2006] using the fixed length of early *P*-wave time window, the PGA-area method [*Lin and Wu*, 2010] will more depend on later *S*-wave arrivals.

[7] Motivated by the deficiencies and merits in these three methods [*Wu et al.*, 1998; *Wu and Zhao*, 2006; *Lin and Wu*, 2010], in this study we used the area of high initial *P*-wave displacement to correlate with the earthquake magnitude. We show that the logarithm of the area enclosed by a P*d* contour

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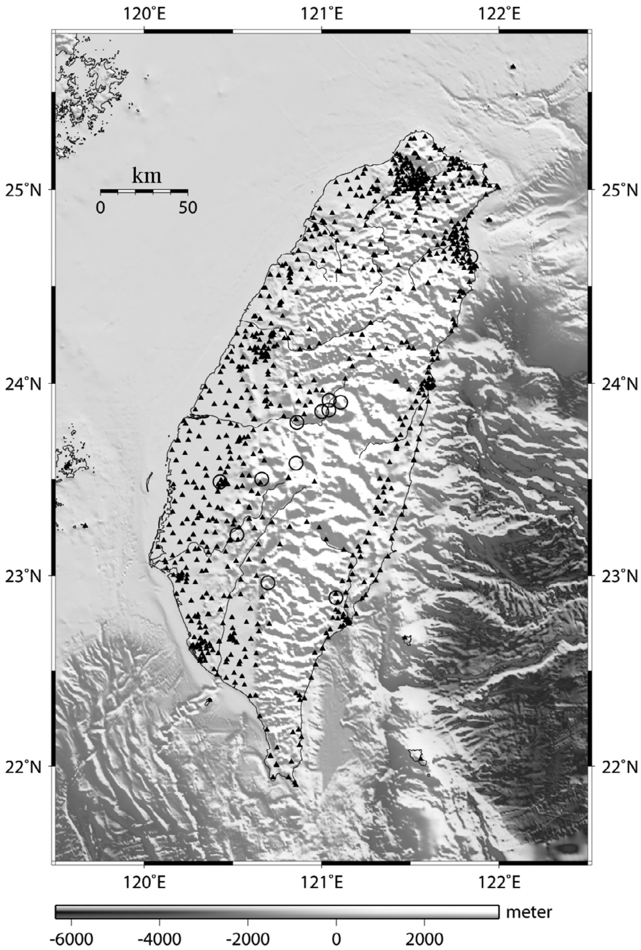

Figure 1. Locations of seismic stations (triangles) of the TSMIP seismic network. The epicenters of the 13 crustal earthquake events with P*d* larger than 0.35 cm recorded by the TSMIP stations are shown as the open circles.

has a linear relation to the corresponding earthquake magnitude similar to using PGA area. Our new proposed P*d*-area method inherits the merit of the PGA-area method [*Lin and Wu*, 2010] in omitting the earthquake location determination and eliminates the deficiency in collecting time of using PGA from later *S*-waves arrivals.

2. Strong Ground Motion Data

[8] In this study we collected the strong ground motion data recorded by the TSMIP stations in the time period from 1993 to 2010. The TSMIP [*Liu et al.*, 1999] operated in non-real-time by CWB consists of over 800 free-field stations densely distributed throughout the Taiwan island as of 2008 (Figure 1). The TSMIP has a station spacing of about 5 km throughout most populated areas. However, station coverage is more limited, and inter-station spacing larger, in the mountain ranges.

[9] The original accelerograms were double integrated to find the displacement and the displacements were filtered using a high-pass recursive Butterworth filter with a corner at 0.075 Hz as empirically suggested by *Kanamori* [2005] and *Wu and Kanamori* [2005a, 2005b, 2008a, 2008b]. The first arrivals were automatically detected from the raw vertical accelerograms by a *P*-wave picker described by *Allen* [1978] as adopted in the current EEW system in Taiwan. The absolute amplitudes of the initial filtered vertical displacement after the first automatically-picked *P*-wave arrival of between 0.35 cm and 1.50 cm are used to make the corresponding contours in this study. In most cases, the contour level amplitudes are exceeded less than 3 seconds after the *P*-wave arrival. If the displacement exceedance occurs after 5 seconds of the initial *P*-wave arrival, the station is not included in this study. Based on the empirical relationships between P*d* and PGV ($\log(PGV) = 0.920\log(Pd) + 1.642$) [*Wu and Kanamori*, 2008a, equation 5], and one between PGV and PGA ($\log(PGA) = 0.595 + 1.069\log(PGV)$) [*Wu et al.*, 2003, equation 12], P*d* values of 0.35 and 1.50 cm will have PGA values of about 80 and 330 Gal, respectively. Each contour contains at least 10 TSMIP stations, and the contours are carefully, manually built in considering of the TSMIP station distribution. Finally, 13 earthquakes with at least 10 stations of P*d* larger than 0.35 cm are selected (Table 1). These 13 earthquakes are all inland, shallow events (Figure 1) of focal depths less than 25 km and moment magnitudes (M_w) in between 5.4 and 6.5. In fact, almost all earthquakes with PGA exceeding 100 Gal have focal depths shallower than 25 km and are located inland or offshore (distance to shoreline <5 km) in Taiwan.

3. Data Analysis and Results

[10] We propose an empirical relationship between the areas inside the P*d* contours and the respective earthquake magnitudes of the form similar to those of *Teng et al.* [1997] and *Lin and Wu* [2010]:

$$M = a\log_{10}(A) + b(Pd) + c, \qquad (1)$$

where A (km^2) is the area enclosed by the contour of P*d* (cm), M is the moment magnitude (M_w), and a, b, c are the empirical coefficients to be determined from a regression analysis. We have adopted the M_w that were reported by the Harvard centroid moment tensor (CMT) project.

[11] Equation (1) can be rewritten in matrix form as

$$\begin{bmatrix} M_1 \\ M_1 \\ \vdots \\ M_{13} \end{bmatrix}_{69\times 1} = \begin{bmatrix} \log_{10} A_1 & Pd_1 & 1 \\ \log_{10} A_2 & Pd_1 & 1 \\ \vdots & \vdots & \vdots \\ \log_{10} A_{69} & Pd_{13} & 1 \end{bmatrix}_{69\times 3} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix}_{3\times 1}, \qquad (2)$$

Table 1. Parameters of the 13 Events

Origin Time	Latitude (N)	Longitude (E)	Depth (km)	M_L	M_w
1993/12/15; 21:49:43	23.213	120.524	13	5.7	5.4
1998/07/17; 04:51:15	23.503	120.663	3	6.2	5.7
1999/09/20; 17:57:16	23.912	121.044	8	6.4	6.3
1999/09/20; 18:03:42	23.797	120.861	10	6.6	6.5
1999/09/20; 18:16:18	23.862	121.041	13	6.7	6.5
1999/09/20; 21:46:38	23.585	120.857	9	6.6	6.4
1999/09/25; 23:52:50	23.854	121.002	12	6.8	6.5
1999/10/22; 02:18:57	23.488	120.428	21	6.4	5.9
2000/06/10; 18:23:29	23.901	121.109	16	6.7	6.4
2005/03/05; 19:06:52	24.655	121.841	6	5.9	5.7
2005/03/05; 19:08:00	24.653	121.798	7	6.0	5.7
2006/04/01; 10:02:20	22.884	121.081	7	6.2	6.1
2010/03/04; 00:18:52	22.962	120.699	23	6.4	6.3

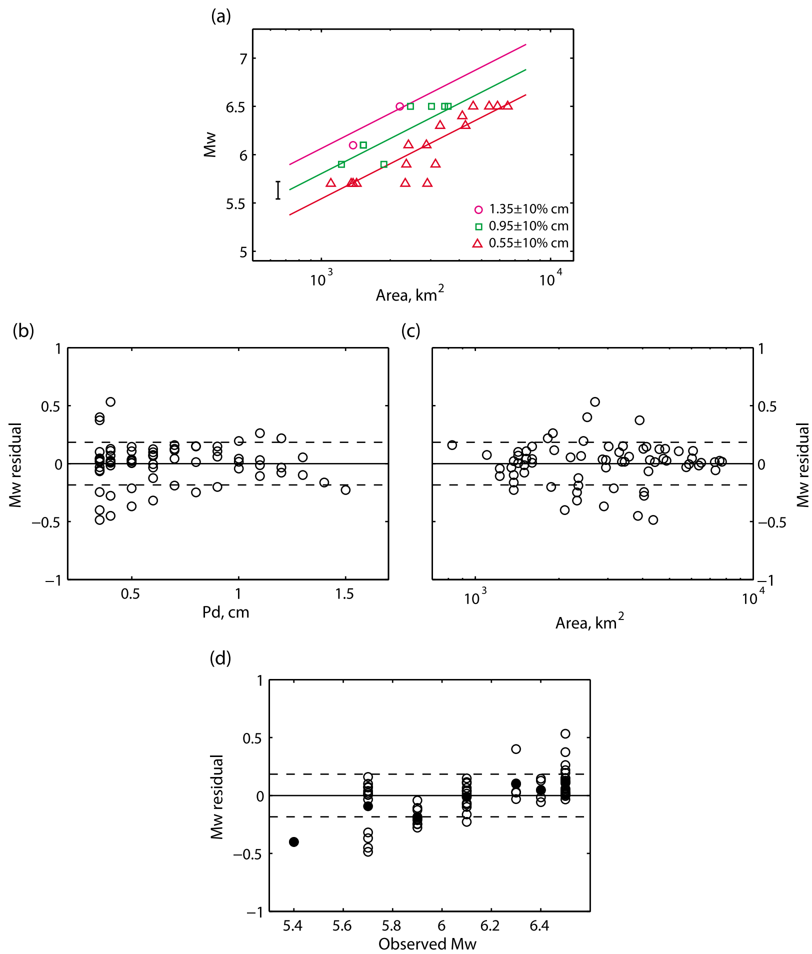

Figure 2. (a) Comparisons between the observed M_w with the predicted magnitudes by equation (3) for the three different Pd values (Pd = 0.55, 0.95, and 1.35 in equation (3)). The height of the bar equals to the standard deviation of 0.18 in equation (3). The M_w residual are plotted against (b) Pd value used to define the area, (c) the area, and (d) the observed M_w. The solid circles in Figure 2d are the average of the M_w residual in the open circles. The dashed lines in Figures 2b, 2c, and 2d represent the level of the standard deviation of 0.18 in equation (3).

where Pd ranges continuously between 0.35 cm (Pd_1) and 1.50 cm (Pd_{13}) as indicated in Figure 2b. Equation (2) presents a typical overdetermined inversion problem and can be viewed as Gm = d. G and d are the data kernel and data vector, respectively. The vector of unknowns (model parameter vector: m) were found through generalized inverse matrix of G (G^{-g}) using singular value decomposition (SVD) [*Menke*, 1984; *Miao and Langston*, 2007]. Using 69 area readings from 13 events recorded by the dense TSMIP stations in Taiwan, we thus obtain a Pd-area relationship as:

$$M_{Pd-A} = 1.207 \log_{10}(A) + 0.651 Pd + 1.566 \pm 0.18. \quad (3)$$

[12] The standard deviation of 0.18 is surprisingly small by using only the early portion of the P-wave and is quite satisfying for practical EEW operation. However, the quantity of the events used in this study (13 events) might question the statistical significance of the Pd-area relationship. Figure 2a compares the observed M_w with the predicted ones by equation (3) for three examples of Pd values (Pd = 0.55, 0.95, and 1.35 in equation (3)) and shows close comparisons except for few of those derived by the contour value of Pd = 0.55. Figure 2b plots the M_w residual against Pd value used to define the area. The M_w residual is defined as the real number between the observed M_w and the calculated M_w ($M_{Wobs} - M_{Wcalc}$). The result of Figure 2b might imply that the

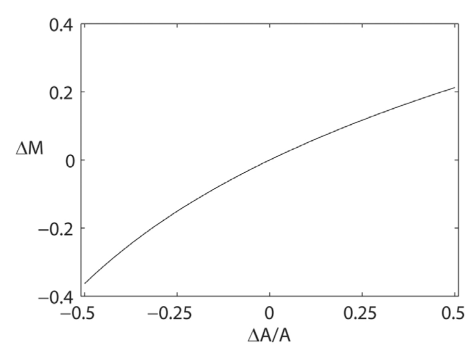

Figure 3. The relationship between the uncertainty in area estimation ($\frac{\Delta A}{A}$) and the resulting variation in magnitude (ΔM) as shown in equation (5).

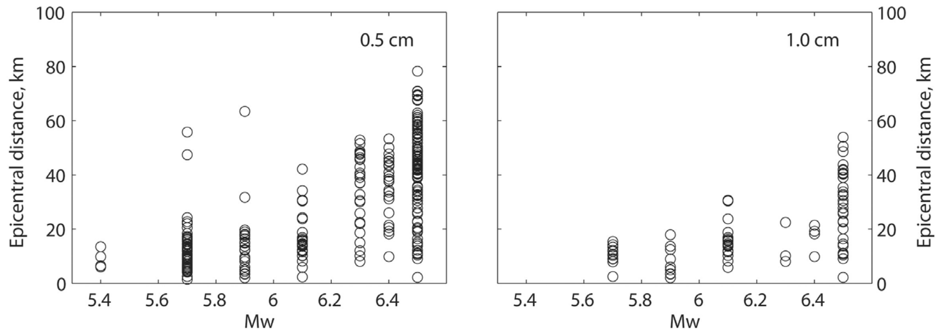

Figure 4. The epicentral distance distribution of the TSMIP stations with P*d* values of 0.5 and 1.0 cm, respectively.

M_w residual tends to be larger for using lower values of P*d* contour than those of using greater ones. However, owing to inequality in the numbers of the events in each P*d* level, this implication suggested by Figure 2b is not completely conclusive. Figure 2c shows that there is no obvious relation between the M_w residual and the area, which could be loosely translated from Figures 2b and 2d. Figure 2d compares the M_w residual with the observed M_w and shows that for a larger magnitude earthquake ($M_w \geq 6.5$) the calculated M_w tends to underestimate the observed M_w implying P*d* saturation.

4. Discussion and Conclusion

[13] The 1999 Chi-Chi earthquake, Taiwan (M_w = 7.6), which is to date the largest recorded and the most disastrous earthquake to have occurred in Taiwan, was not used in the regressions. The standard deviation of 0.18 in equation (3) would significantly increase to 0.52 if the data of the 1999 Chi-Chi earthquake is used. The P*d*-area relationship (not shown here) of including the 1999 Chi-Chi earthquake consistently underestimated the Chi-Chi earthquake's magnitudes by 0.5 to 1.4 units suggesting the saturation effect of large earthquakes [*Wu et al.*, 2006]. The largest magnitude used in this study is of 6.5 in M_w. Therefore, conservatively speaking, our proposed method should be applicable to earthquakes of magnitudes at least up to 6.5 as also pointed out in studies [*Kanamori*, 2005; *Wu and Zhao*, 2006; *Zollo et al.*, 2006] using the initial *P*-wave to estimate magnitude. Even an inland, shallow earthquake with a magnitude of 6.5 occurring near populated areas is capable of causing severe damages. In the case of a larger than M_w = 6.5 earthquake, our proposed method could at least rapidly provide a lower bound of the earthquake magnitude and the PGA-area method [*Lin and Wu*, 2010] might progressively offer more accurate estimations as source rupture is more complete.

[14] One source of the magnitude discrepancy estimated by equation (3) is the uncertainty in the area estimation (*A* in equation (3)), for example, caused by the uneven station sampling, source radiation pattern, and site effects. Equation (3) can be rewritten as equation (4) if a deviation in A (ΔA) is introduced, and eventually, ΔM and ΔA in equation (4) can be rearranged as derived in equation (5).

$$M + \Delta M = 1.207 \log_{10}(A + \Delta A) + 0.651Pd + 1.566. \tag{4}$$

$$\Delta M = \left(\frac{1.207}{\ln 10}\right) \ln\left(1 + \frac{\Delta A}{A}\right). \tag{5}$$

[15] Figure 3 reveals the relationship between ΔM and $\frac{\Delta A}{A}$ as shown in equation (5). As indicated in Figure 3, the variations in ΔM by varying $\frac{\Delta A}{A}$ between −0.25 and 0.25 are within the standard deviation of 0.18 in equation (3). Positive and negative values of $\frac{\Delta A}{A}$ represent overestimate and underestimate on area, respectively.

[16] Figure 4 summarizes the epicentral distances of the recording stations with P*d* values of 0.5 and 1.0 cm, respectively. In general, for a larger magnitude earthquake a same level of P*d* would propagate to a greater epicentral distance. Stations with the epicentral distances of 80 and 55 km are the furthest stations recording P*d* values of 0.5 and 1.0 cm, respectively, for an M_w = 6.5 earthquake. Assuming a *P*-wave velocity of 6.0 km/s, the travelling times over distances of 80 and 55 km are of about 13 and 9 seconds, respectively. By considering of the time needed for recording a sufficient length of the waveforms and the system processing time, the reporting time will be most less than 19 and 15 sec for P*d* values of 0.5 and 1.0 cm, respectively.

[17] The major requirement to effectively apply our proposed P*d*-area method in a real-time, automatic manner is to have a real-time seismic network with an applicable station density since the accuracy of the area estimation mainly depends on the station density. The EEW research group at the National Taiwan University has developed a commercial MEMS-type (Micro Electro Mechanical Systems) accelerometer specifically designed for EEW purposes. This EEW accelerometer is cost-saving, and is capable for recording near-field, high-frequency ground motions. Our rapid magnitude estimation method will become more practical and accurate once the cost-saving EEW accelerometer has been densely installed in Taiwan.

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