# *A Fast Magnitude Estimation for the 2011 $M_w$ 9.0 Great Tohoku Earthquake*

**by Ting-Li Lin and Yih-Min Wu**

## INTRODUCTION

After the occurrence of a large earthquake, rapid reporting is useful and necessary for resource dispatch management and quick damage assessment. In practice, a ground-motion prediction or a shaking-intensity estimation can be realized once the location and magnitude of an earthquake become available. Rapid availability of event information can also fulfill inquiries and alleviate the misgivings of the public and media.

Using the traditional travel-time method, rapid earthquake location is a simple task once a few *P*-wave arrivals are known. In contrast, a quick and accurate estimate of earthquake magnitude is a nontrivial problem, especially for an extremely large regional event. For a teleseismic-determined moment magnitude, it may take tens of minutes for the propagation of teleseismic waves. For a method using first-arrival *P* waveforms of broadband seismograms for moment magnitude determinations (Tsuboi *et al.*, 1995, 1999), the reporting time for moderate-sized earthquakes still takes several minutes. In addition, common high-gain broadband stations will suffer a severe amplitude-clipping problem from regional large earthquakes.

The $M_w$ 9.0 (Global Centroid Moment Tensor [CMT]) Tohoku-Chiho Taiheiyo-Oki (Tohoku) earthquake that occurred on 11 March 2011 was unprecedented in size since Japan started modern instrumental recordings 130 years ago. The megathrust Tohoku earthquake ruptured the Pacific–North American (Okhotsk) plate boundary off the Pacific coast of northeastern Japan.

The work presented here employed the empirical method of Wu and Teng (2004) for quick $M_w$ determinations of crustal earthquakes in Japan using time integration over the strong-shaking duration for absolute values of acceleration records.

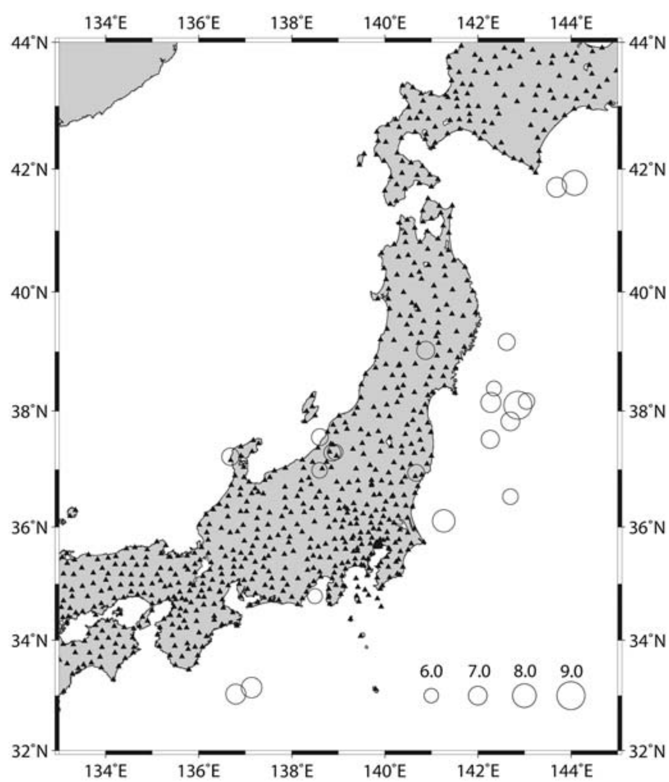
In the study of Wu and Teng (2004), the maximum magnitude was for the 1999 $M_w$ 7.6 (Global CMT) Chi-Chi earthquake in Taiwan. In this work, the largest event was the 2011 $M_w$ 9.0 Tohoku earthquake in Japan. We show that the method of Wu and Teng (2004) can be applied for large crustal earthquakes in Japan and even for extremely large events such as the 2011 Tohoku earthquake—one of the five largest earthquakes that have occurred in the modern era—without a saturation problem.

## METHOD AND DATA

In this work, we only provide brief descriptions for the quick $M_w$ determination. For more information, readers should refer to the article of Wu and Teng (2004). First, we define the absolute-value acceleration integral ($\sqrt{Es}$) as follows:

$$\sqrt{Es} = \int_{T_p}^{T_e} \sqrt{V^2 + N^2 + E^2}\,\mathrm{d}t, \qquad (1)$$

where $V$, $N$, and $E$ are the vertical, north–south, and east–west acceleration components in gal, respectively. $T_p$ is the *P*-wave arrival time, and $T_e$ is the time of the end of strong




▲ **Figure 1.** The locations of the seismic stations (triangles) within the NIED K-NET seismic network. The epicenters of the 21 crustal earthquake events are shown with open circles scaled by magnitude.

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**Table 1**
**Parameters of the 21 Events Used in This Study**

| Origin Time (yyyy/mm/dd UTC) | Latitude (°N) | Longitude (°E) | Depth (km) | $M_w$ | No. of $\sqrt{Es}$ |
|---|---|---|---|---|---|
| 9/25/2003 19:50 | 41.780 | 144.079 | 42 | 8.3 | 223 |
| 9/25/2003 21:08 | 41.707 | 143.695 | 21 | 7.3 | 172 |
| 10/31/2003 01:06 | 37.829 | 142.700 | 33 | 7.0 | 44 |
| 9/5/2004 10:07 | 33.030 | 136.801 | 38 | 7.2 | 287 |
| 9/5/2004 14:57 | 33.144 | 137.142 | 44 | 7.4 | 286 |
| 10/23/2004 08:56 | 37.290 | 138.870 | 20 | 6.6 | 211 |
| 10/23/2004 09:34 | 37.303 | 138.933 | 10 | 6.3 | 174 |
| 8/16/2005 02:46 | 38.151 | 142.208 | 43 | 7.2 | 259 |
| 3/25/2007 00:42 | 37.221 | 136.686 | 11 | 6.7 | 189 |
| 7/16/2007 01:13 | 37.557 | 138.610 | 17 | 6.6 | 208 |
| 6/13/2008 23:43 | 39.030 | 140.881 | 8 | 6.9 | 178 |
| 7/19/2008 02:39 | 37.521 | 142.265 | 32 | 6.9 | 78 |
| 12/20/2008 10:29 | 36.531 | 142.700 | 12 | 6.3 | 12 |
| 8/10/2009 20:07 | 34.786 | 138.499 | 23 | 6.2 | 232 |
| 3/9/2011 21:23 | 38.172 | 143.045 | 10 | 6.5 | 60 |
| 3/11/2011 05:46 | 38.104 | 142.861 | 24 | 9.0 | 413 |
| 3/11/2011 06:15 | 36.108 | 141.265 | 39 | 7.8 | 219 |
| 3/11/2011 11:37 | 39.170 | 142.619 | 24 | 6.6 | 149 |
| 3/11/2011 18:59 | 36.986 | 138.598 | 8 | 6.3 | 176 |
| 3/27/2011 22:24 | 38.384 | 142.346 | 21 | 6.2 | 107 |
| 4/11/2011 08:16 | 36.946 | 140.673 | 12 | 6.6 | 247 |
| | | | | | Total = 3,924 |

Parameters are compiled from the Global CMT and the NIED catalogs

shaking. $T_e$ is defined as the point when the vector sum of the three-component accelerations ($\sqrt{V^2 + N^2 + E^2}$) drops continuously for 5 s below 20% of the maximum. The purpose of the use of acceleration records instead of velocity records for determining $\sqrt{Es}$ is to reduce the effects of the long-period waves by the excitation of surface waves and the accumulation of long-period noise (Wu and Teng, 2004). $\sqrt{Es}$ has a unit of velocity; thus, $Es$ is proportional to the wave energy. $\sqrt{Es}$ may be interpreted as the total effective shaking embodied in the waveforms. By means of an attenuation relationship using the hypocentral distance, $\sqrt{Es}$ is then empirically correlated with the magnitude. Unlike common ground-motion attenuation relationships, $\sqrt{Es}$ is used as the decaying amplitude. The linear regression model is expressed as follows:

$$\log\left(\sqrt{Es}\right) = A + BM_w + CR + D\log(R), \qquad (2)$$

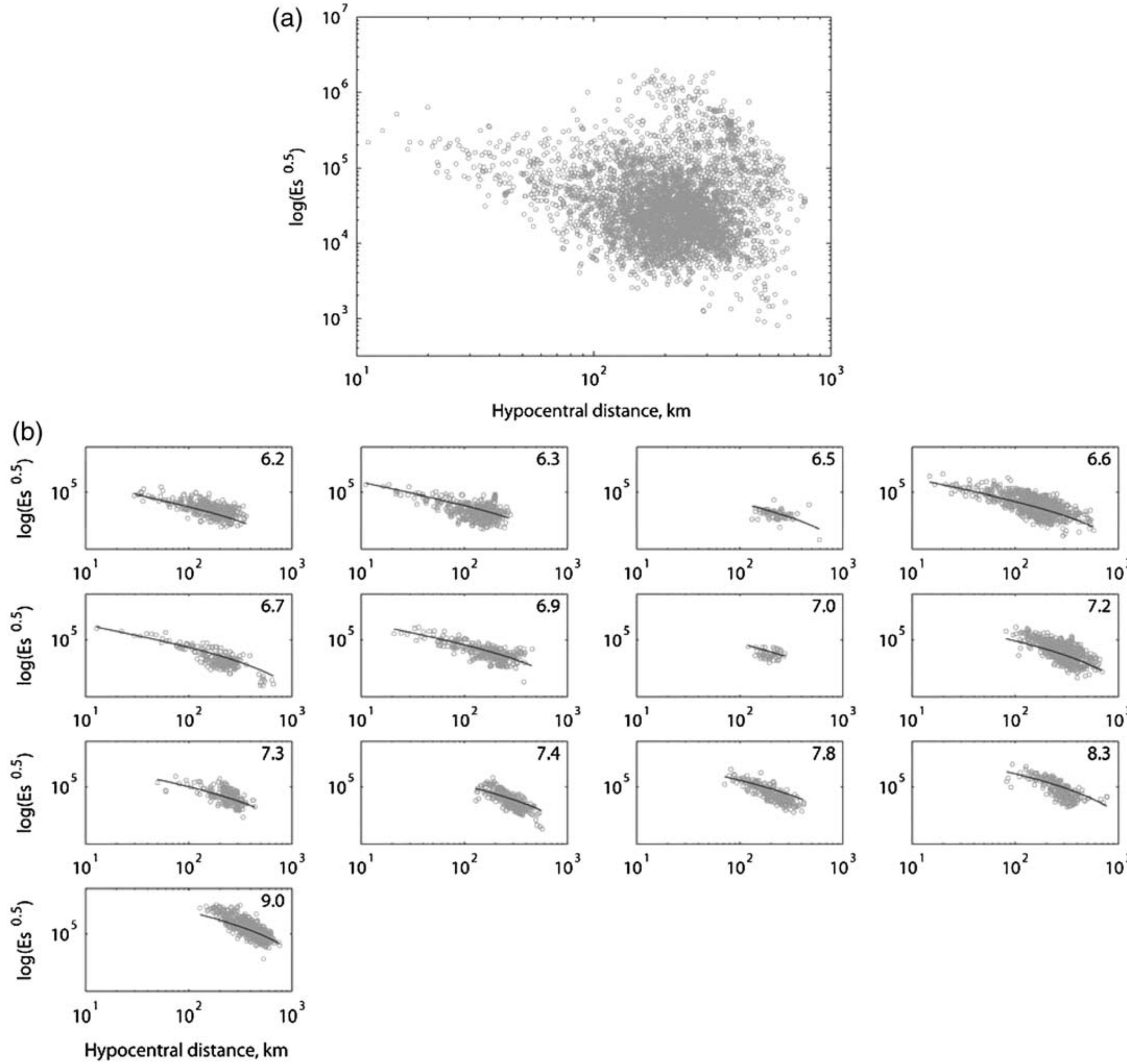
where $R$ is the hypocentral distance, and the empirical coefficients $A$, $B$, $C$, and $D$ can be determined from regression analysis using singular value decomposition (Menke, 1984; Miao and Langston, 2007).

Acceleration data are acquired from the K-NET stations of the National Research Institute for Earth Science and Disaster Prevention (NIED), Japan (Aoi *et al.*, 2009). K-NET consists of more than 1,000 free-surface, strong-motion accelerometers (Fig. 1). K-NET uniformly covers all of Japan, with an average station-to-station distance of approximately 20 km. Twenty-one large ($M_j$≥6.5) and shallow (focal depth < 45 km) earthquakes (Table 1) between 1996 and 2011 have been chosen in this study. These earthquakes are the main targets of earthquake rapid-reporting operation based on their close distances to the populated areas and large magnitudes. $M_j$, the magnitude parameter for the K-NET online data acquisition system, is the local magnitude based on maximum amplitudes of seismograms, as defined and calculated by the Japan Meteorological Agency. A total of 3,924 $\sqrt{Es}$ values from 21 events were used to derive the wave-energy attenuation relationship (equation 2). Figure 2 provides the values of $\sqrt{Es}$ versus the hypocentral distance and clearly indicates the attenuation of $\sqrt{Es}$.

## RESULTS

The resulting $\sqrt{Es}$ attenuation relationship is

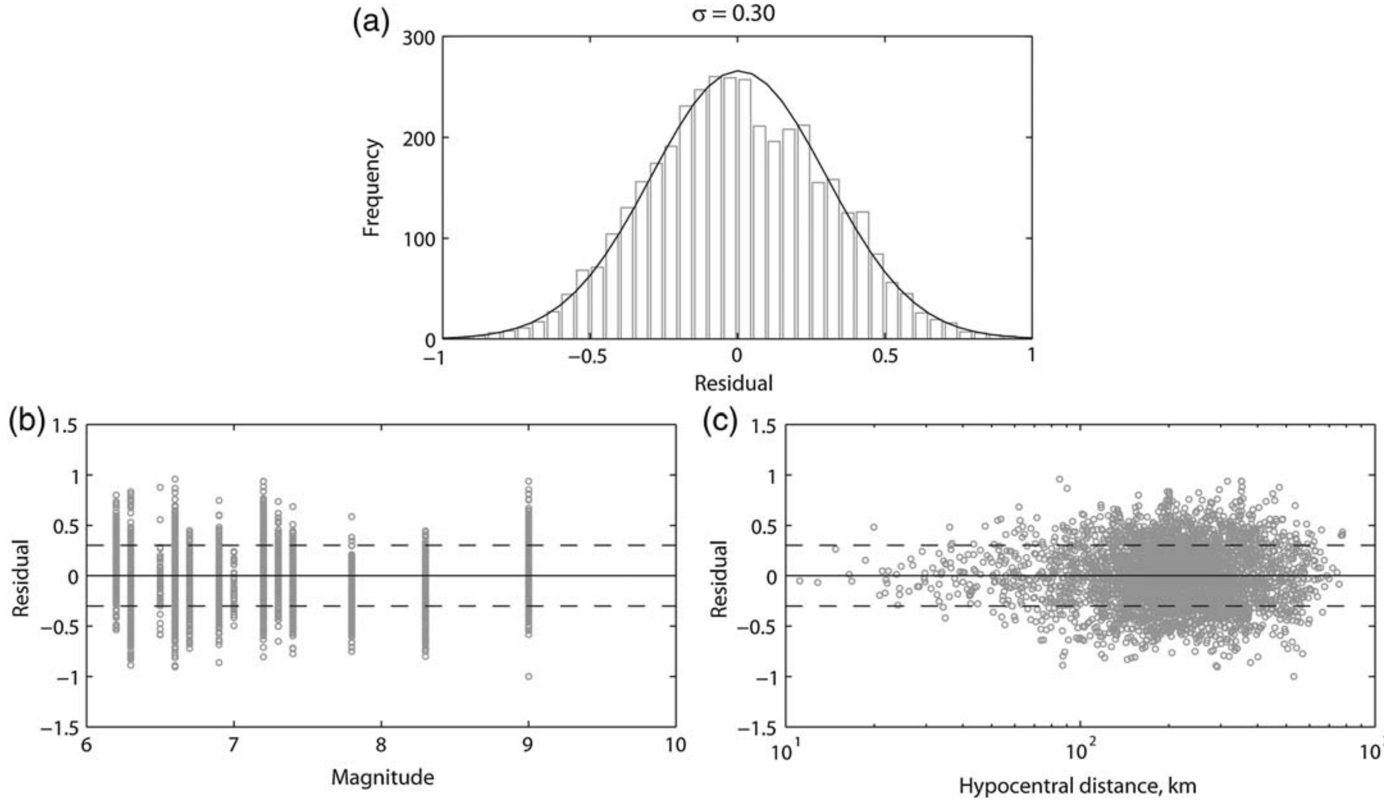
$$\log(\sqrt{Es}) = 0.7501 + 0.5755M_w - 0.0009R - 0.9294\log(R) \pm 0.296. \qquad (3)$$




▲ **Figure 2.** Effective shaking versus hypocentral distance. (a) $\sqrt{Es}$ for 3,924 values and (b) $\sqrt{Es}$ separated by magnitudes. Solid lines represent the predicted $\sqrt{Es}$ as computed by equation (3). The reported $M_w$ values are indicated in the upper right corner of each plot.

The predicted values of $\log(\sqrt{Es})$ from equation (3) are plotted against the observed values in Figure 2. In general, the predicted $\log(\sqrt{Es})$ values are in good agreement with the observations. Figure 3a shows the distribution of the $\log(\sqrt{Es})$ residuals (observed minus predicted values), which follows a normal distribution with a standard deviation ($\sigma$) of 0.296, indicating no systematic bias. According to the data, $\log(\sqrt{Es})$ residuals do not show any particular trend with magnitude or hypocentral distance (Fig. 3b, c).

Using equation (3), one can quickly estimate $M_w$ by simply inverting equation (3) once earthquake location and *Es* have been determined. Figure 4 summarizes the inverted $M_w$, by applying equation (3), for all 21 study events as functions of the hypocentral distance (Fig. 4a) and the number of recordings (Fig. 4b). The mean estimated $M_w$ fits the reported $M_w$ remarkably well even for the 2011 $M_w$ 9.0 Tohoku earthquake. The estimated $M_w$ values, by using the recordings of those with shorter hypocentral distance (Fig. 4a) and fewer recordings (Fig. 4b), even provide close initial estimates for the magnitudes. Without using data from the 2011 $M_w$ 9.0 Tohoku earthquake (not shown here) in the regression analysis, the inverted $M_w$ for the 2011 $M_w$ 9.0 Tohoku




▲ **Figure 3.** (a) The distribution of the residuals between the observed and predicted (equation 3) effective shaking ($\sqrt{Es}$). The variance of the residual distribution was used for the error estimation (⊠). Residuals are plotted against (b) moment magnitude and (c) logarithms of the hypocentral distance.

earthquake using the $\sqrt{Es}$ attenuation relationship is approximately 9.51, suggesting that there is no saturation problem (Fig. 5).

## DISCUSSION AND CONCLUSIONS

We follow the method of Wu and Teng (2004) to determine the magnitude of the 2011 $M_w$ 9.0 Great Tohoku Earthquake. Our work is an empirical approach for near real-time moment magnitude determinations for large ($M_w$>6.5) earthquakes (and is even appropriate for extremely large events such as the Great Tohoku Earthquake of 2011) without a saturation problem.

A large event with large rupture dimensions, such as the 2011 $M_w$ 9.0 Tohoku earthquake, likely consists of several major energy bursts, suggesting a number of major dislocations or strong asperities over the entire rupture volume. Therefore, using the first few seconds of the waveform from an initial nucleation rupture for estimating the eventual magnitude, which is the concept behind the earthquake early warning (EEW) system, may be difficult for extremely large earthquakes. In the case of the 2011 Tohoku earthquake, the EEW estimated magnitude from the amplitude and frequency content (Hoshiba and Iwakiri, 2011) is smaller than the reported magnitude of $M_w$ 9.0. Our results indicate that the effective shaking ($\sqrt{Es}$) method of Wu and Teng (2004) provides an excellent magnitude estimation for the 2011 Tohoku earthquake, without a saturation problem. The effective shaking method, although it may consume too much data collection time for use in EEW, is adequate for earthquake rapid reporting and tsunami early warning. The processing time of the teleseismic-determined $M_w$ is way too long for earthquake rapid reporting in most cases.

Applicability of the effective shaking ($\sqrt{Es}$) method depends on the density of the recording seismic network. A dense seismic network can take advantage of the averaging scheme to dilute the effect of abnormal amplitude recordings caused by issues such as (1) near-field, high-frequency spikes; (2) rupture directivity; (3) soft-soil conditions; (4) basin effects; (5) near-source effects; and (6) hanging-wall effects. One of the reasons that the method of effective shaking provides good estimates of earthquake magnitudes in Japan is the high density of K-NET stations.

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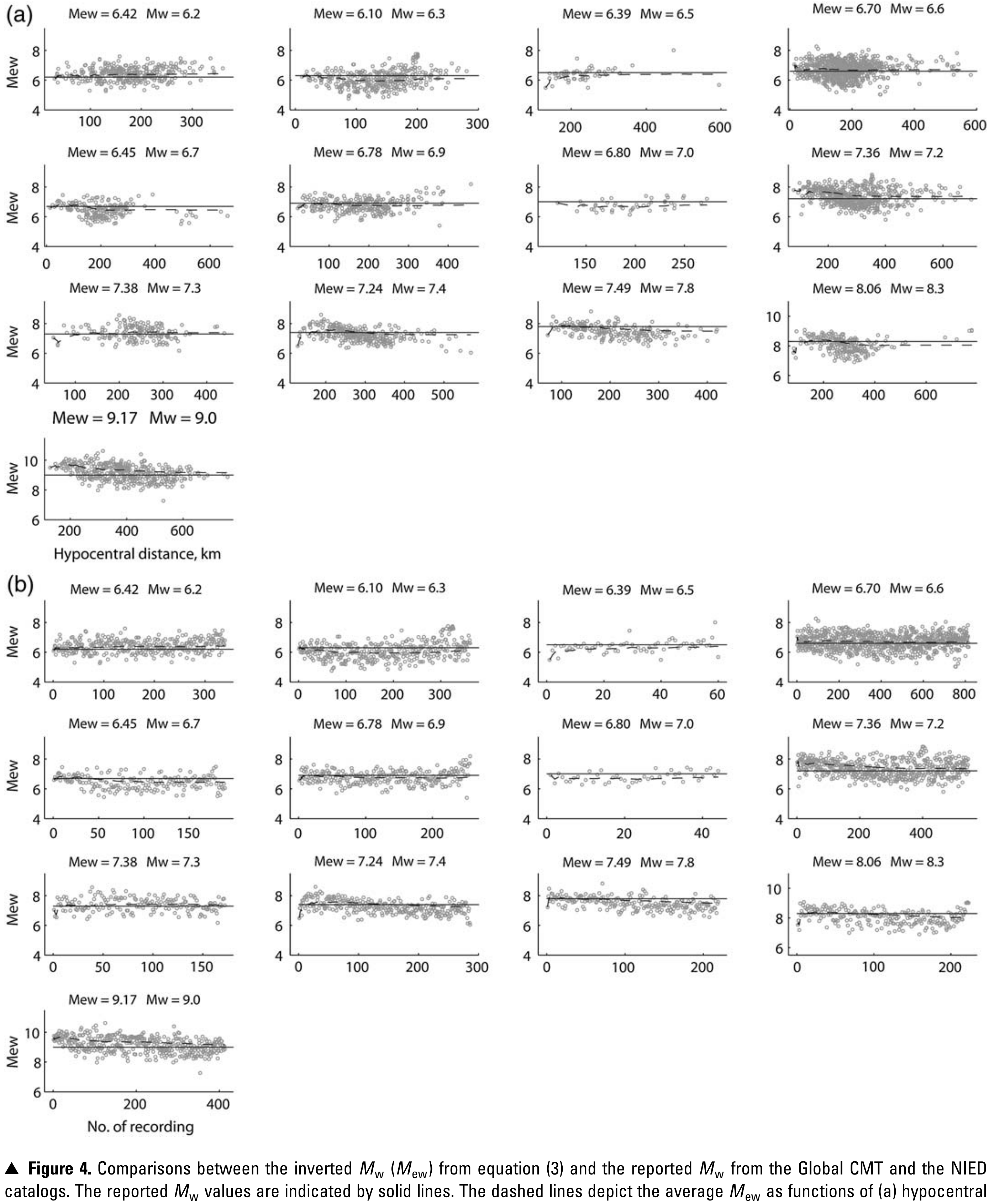
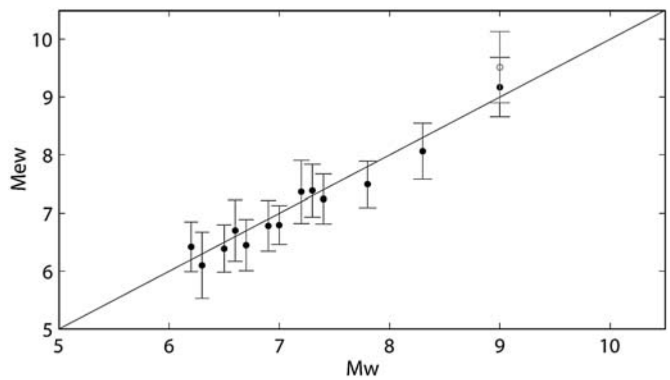



▲ **Figure 4.** Comparisons between the inverted $M_w$ ($M_{ew}$) from equation (3) and the reported $M_w$ from the Global CMT and the NIED catalogs. The reported $M_w$ values are indicated by solid lines. The dashed lines depict the average $M_{ew}$ as functions of (a) hypocentral distance and (b) number of recording.




▲ **Figure 5.** The final averaged $M_{ew}$ values are plotted against a 45° line of $M_{ew} = M_w$. The individual standard deviation of each magnitude was obtained in Figure 4. The gray error bar indicates the estimated $M_w$ for the 2011 $M_w$ 9.0 Tohoku earthquake by the $\sqrt{Es}$ attenuation relationship, without using the data of the 2011 $M_w$ 9.0 Tohoku earthquake.

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