



# Evolution of the temporal multifractal scaling properties of the Chiayi earthquake ( $M_L = 6.4$ ), Taiwan

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## ABSTRACT

Variations in the fractal dimension of earthquakes have been suggested to be a precursor of a large earthquake. However, the physical characteristics and seismicity are always different along a large fault system, and it is difficult to segment a large fault for further investigating. Therefore, the fractal dimension of earthquakes on a given fault only reflects the average seismic characteristics of the area and may be unrelated to precursory activity. In this paper, the evolution of the temporal generalized fractal dimension ( $D_q$ ) of a seismic cluster within a small fault system associated with the Chiayi earthquake ( $M_L = 6.4$ ), Taiwan, is investigated. The earthquakes of this sequence are confined in small source volume and reflect the behavior of the local fault system.

Our results show that the  $D_q$  curve of the background stage is smooth with a low multifractal degree (0.36) and the seismicity is nearly monofractal in the Chiayi region. During the foreshock stage, the seismicity becomes active such that not only the  $D_q$  curve exhibits greater variability especially for the steep slope of  $D_q$  curve at  $q = 0$  but also the temporal fractal dimension changes from nearly monofractal to multifractal. In the aftershock stage, the fluctuation of  $D_q$  is large and the multifractal degree is up to 1.0. The temporal multifractal property becomes more pronounced as well. Finally, when the seismicity returns to the background stage, the  $D_q$  curve becomes smooth and nearly monofractal again. Therefore, the fluctuations in the  $D_q$  spectrum of an earthquake cluster associated with a large earthquake not only give a clear picture of the temporal alterations in the seismogenic regimes but also illustrate their variability through time for a large earthquake. In addition, the variation of the slope of  $D_q$  curve at  $q = 0$  in the foreshocks may be used as a precursor of a large earthquake.

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## 1. Introduction

Mandelbrot (1967) proposed a new concept of the geometric dimension, the fractal dimension, to estimate the complexity of the geometry of natural phenomenon. This concept has been used to estimate the fractal dimensions of fault traces and the distributions of epicenters and hypocenters (Aviles et al., 1987; Guo and Ogata, 1995; Henderson et al., 1999; Okubo and Aki, 1987; Öncel et al., 1996). Multifractal is one kind of fractal dimension defined as a union of several monofractals (Mandelbrot, 1989). Nowadays, multifractal has become one of the scaling laws to characterize the dynamic system, for example, the seismogenic regime.

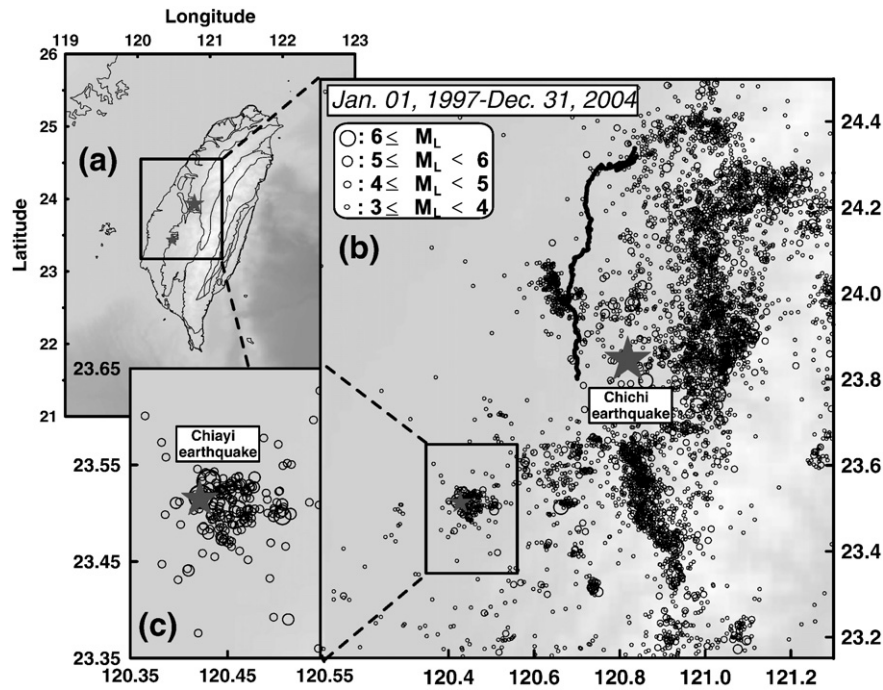
Spatial multifractal properties of earthquakes have been used to analyze the spatial distributions of earthquakes (Hirata and Imoto, 1991; Mittag, 2003; Nakaya and Hashimoto, 2002). In terms of the temporal fractal analysis of earthquakes, the fractal dimension of

the temporal distribution of earthquakes represents the clustering strength (Papadopoulos and Dedousis, 1992). The temporal multifractal variation of seismicity implies the variation of the degree of clustering over time (Godano and Caruso, 1995). In addition, the temporal multifractal variations of earthquake swarms mean the changes in the seismogenic regime (Mittag, 2003).

Hirabayashi et al. (1992) analyzed the temporal multifractal of seismicity in California, Japan and Greece. They concluded that when a large earthquake occurs, the fluctuations of temporal multifractal properties of earthquakes change and become more violent. They defined two types of multifractal spectra – steep type and smooth type. According to the spatial fractal analysis (Godano et al., 1999; Rossi, 1990), the epicenters of earthquakes will cluster before and after a large main shock. However, the physical characteristics and seismicity are always complex within a large fault system, so the fractal geometries of the seismicities along the San Andreas fault (Okubo and Aki, 1987), Sumatra fault (Sukmono, 2001) and Chelungpu (Chi-Chi earthquake) fault (Chang et al., 2007) are different. It is difficult to segment a large fault for further investigation. Hence, the multifractal variations of seismicity only reflect the average effect of seismicity within a large fault system.

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**Fig. 1.** Seismicity in Central Taiwan and the location of the study area. (a) Taiwan is a collision zone of the Asian-Europe plate and the Philippine plate. (b) Distributions of epicenters in Central Taiwan from January 1, 1997 to December 31, 2004. The big star sign indicates the location of the Chichi earthquake, and the small star sign indicates the location of the Chiayi earthquake. (c) The study area from 120.35°E to 120.55°E and from 23.35°N to 23.65°N ( $0.2^\circ \times 0.3^\circ$ ). The star sign indicates the location of the Chiayi earthquake.

In this study, we analyze the multifractal variation of seismicity within a small fault system where the earthquakes are confined in small source volume and reflect the behavior of the fault system (Wen et al., 2008) to test whether the temporal multifractal fluctuation of the seismicity can show the evolvement of a large earthquake.

## 2. Tectonic setting and data

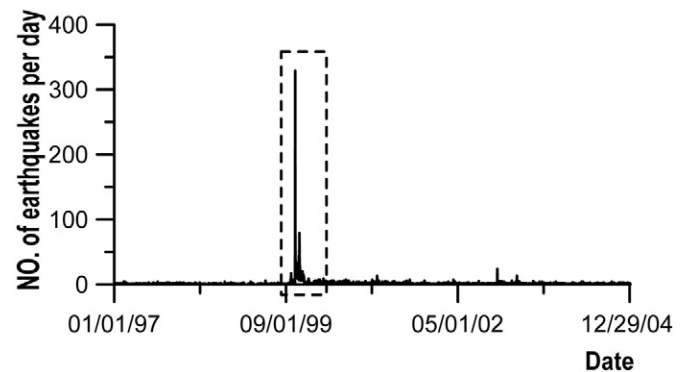
On October 22, 1999, an earthquake of magnitude 6.4 occurred in Chiayi, Taiwan. Hereafter, this earthquake is referred to as the Chiayi earthquake. The earthquake resulted in 5 deaths and 25 injured. Eighteen buildings were destroyed and 165 buildings were damaged by the earthquake. Focal solution indicated that the Chiayi earthquake is resulted from compressional rupture and displacement along a reverse (back-thrust) fault (Wen et al., 2008). Approximately one month before the Chiayi earthquake, the catastrophic Chichi earthquake ( $M_L = 7.3$ ) struck central Taiwan on September 20, 1999. The epicenter of the Chiayi earthquake is located about 55 km south-west of the Chichi earthquake. Using the method of Coulomb failure stress change, Chang and Wang (2006) showed that the static stress changes transferred from the Chichi earthquake may have triggered the Chiayi earthquake.

High quality seismic data of the Chiayi earthquake was recorded from January 1, 1997, to December 31, 2004 via the dense seismic network set up by the Central Weather Bureau (CWB) of Taiwan. Seismicity within Central Taiwan and the study area is shown in Fig. 1. It is clear that the Chiayi earthquake cluster is isolated from the cluster of Chichi earthquake or other clusters. The Chiayi earthquake cluster is centered in the  $0.2^\circ \times 0.3^\circ$  rectangular zone (longitude: 120.35°E to 120.55°E, latitude: 23.35°N to 23.65°N). As the magnitudes and locations of the low magnitude earthquakes are unreliable, a threshold of the local earthquake magnitude scale of 2 ( $M_L \geq 2$ ) is used in this study.

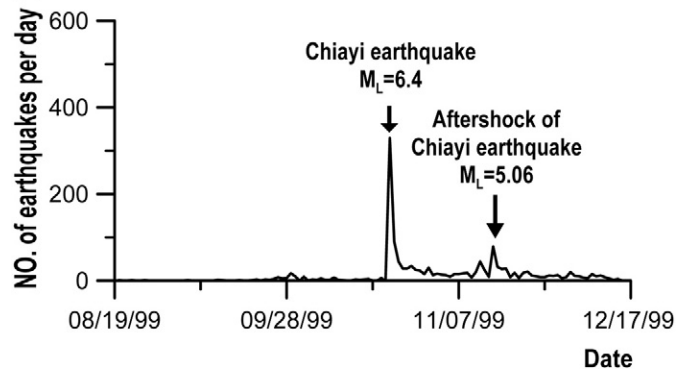
Totally 1538 earthquakes of  $M_L \geq 2$  were identified from the record. The temporal distribution of the Chiayi earthquake cluster is shown in Fig. 2(a). Three large earthquakes from August 20 to December 15, 1999, are of interest: the Chichi earthquake, Chiayi earthquake, and the aftershock ( $M_L = 5.06$ ) of the Chiayi earthquake (Fig. 2(b)). As the Chichi earthquake occurred on September 20, 1999, the seismic activity

in the Chiayi area became more active since that event. The largest aftershock ( $M_L = 6.0$ ) of the Chiayi earthquake occurred about five hours after the main shock, and the second large aftershock ( $M_L = 5.08$ )

### (a) 01/01/97 to 12/31/04



### (b) 08/20/99 to 12/15/99



**Fig. 2.** (a) Temporal distribution of the Chiayi earthquake cluster from January 1, 1997 to December 31, 2004. (b) The interval expanded from the dashed interval in (a). Two large earthquakes are identified during the period from August 20, 1999 to December 15, 1999.

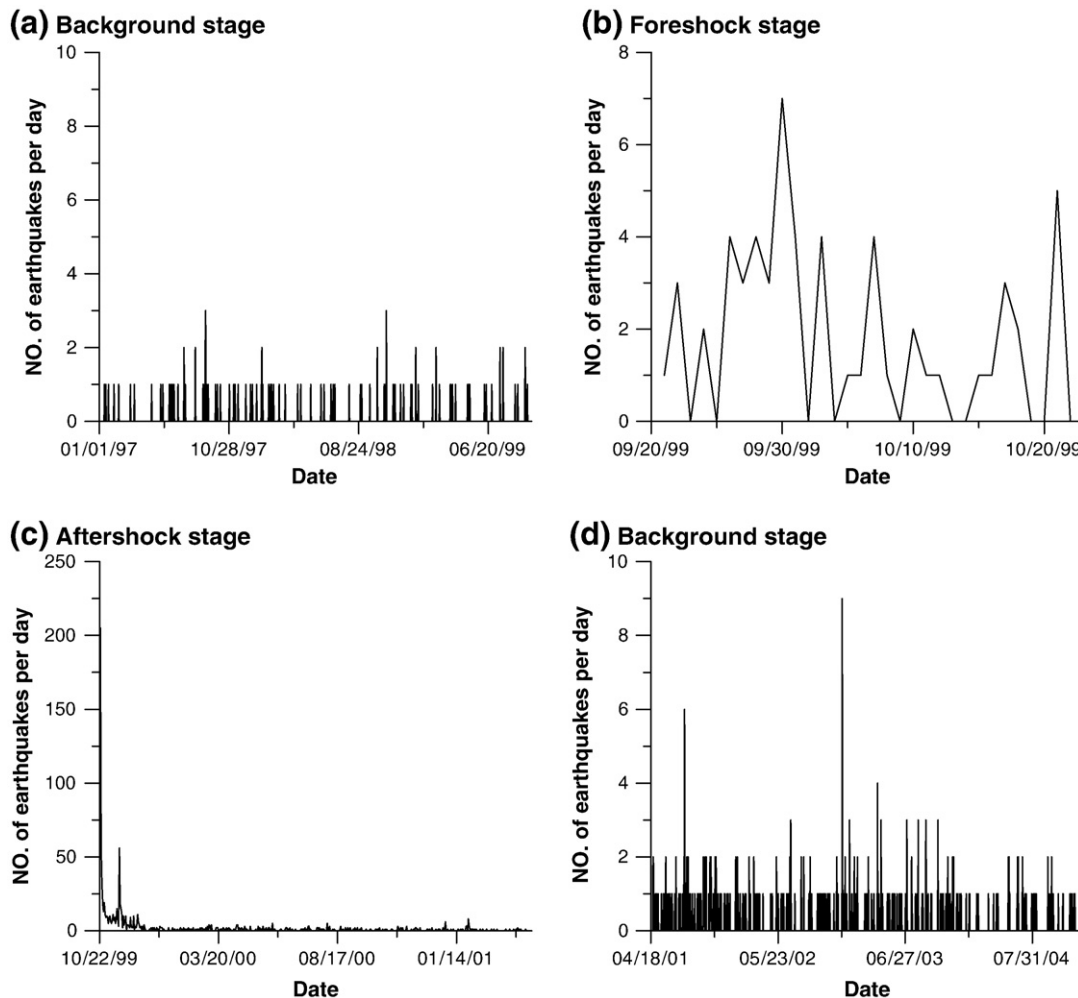
occurred in the next day (October 23, 1999). A large aftershock ( $M_L = 5.06$ ) occurred on November 15, which accompanied a remarkable increase of the number of earthquakes at the same day. The seismic activity in the Chiayi area became stabilized after April 18, 2001. Therefore, four stages of temporal seismicity can be determined for this area. The temporal and spatial distributions of the earthquakes for the four stages are shown in Figs. 3 and 4, respectively. The  $b$  values obtained from the Gutenberg–Richter plots for each of the four stages are shown in Fig. 5.

Stage 1 extends from January 1, 1997 to September 20, 1999 and represents the background stage. During this stage, the average number of the earthquakes per day is less than 1 (Fig. 3(a)), and the epicenters distribute randomly (Fig. 4(a)). The  $b$  value is almost equal to 1 ( $b = 0.97$ ), shown in Fig. 5(a). After the Chichi earthquake, the seismicity in the Chiayi area became more active (Fig. 3(b)), and the epicenters of earthquakes in the Chiayi area are getting cluster (Fig. 4(b)). These earthquakes could be considered as the foreshocks before the Chiayi earthquake. This period is referred to as the Stage 2 and extends from September 20 to October 22, 1999. The average number of earthquakes per day in this stage is greater than 1, and the maximum number of earthquakes in a day is 7. The  $b$  value is lower ( $b = 0.83$ , Fig. 5(b)) than those in the background stages. The Stage 3 extends from October 22, 1999 and ends on April 18, 2001. This stage is determined to be the aftershocks of the Chiayi earthquake. It is seen from Fig. 3(c) that the number of the earthquakes

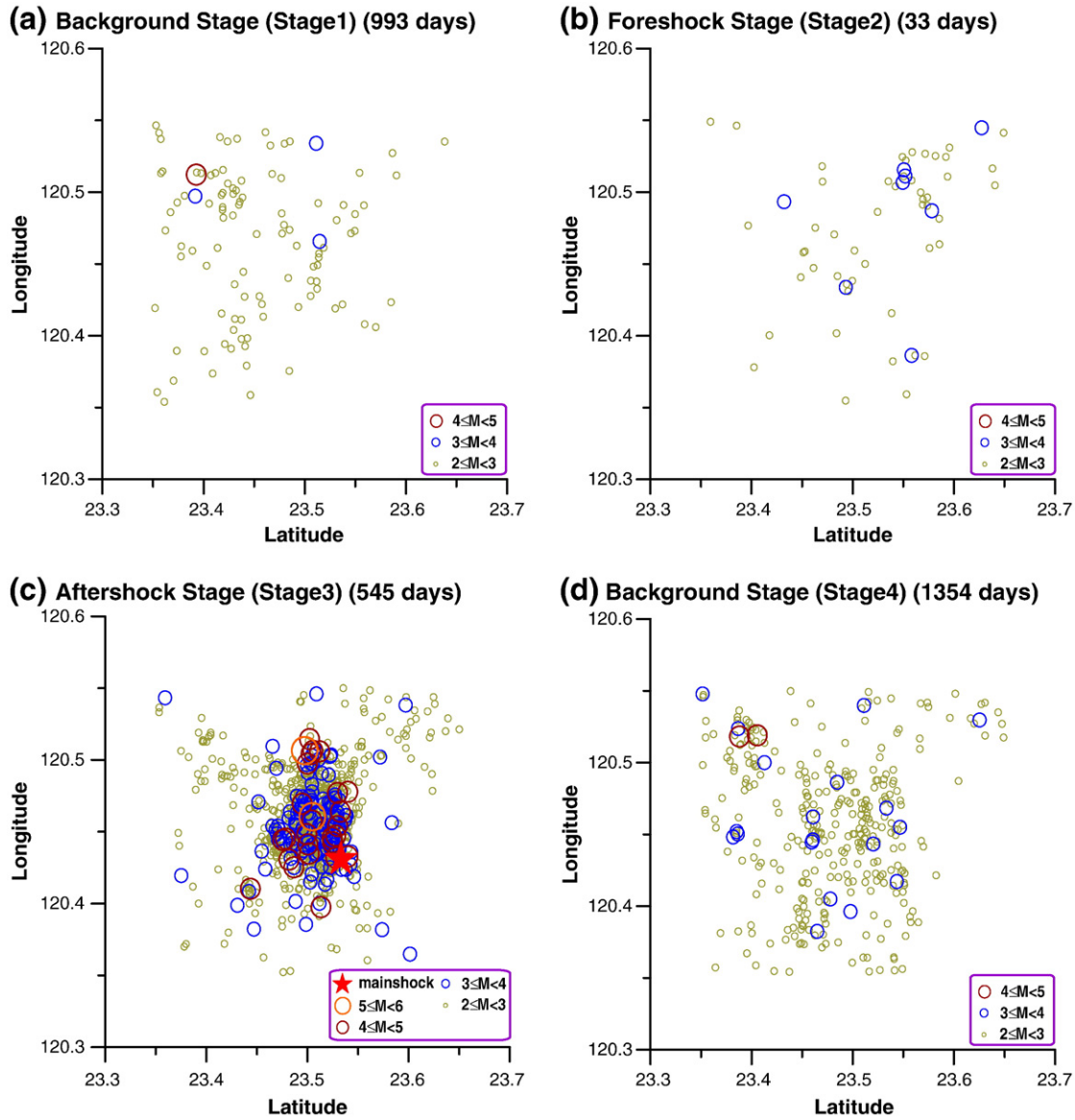
per day varies dramatically. The epicenters get more clustering in space (Fig. 4(c)) and the  $b$  (0.81) value is still low (Fig. 5(c)). Seismicity approaches background levels in the Stage 4 from April 18, 2001, to December 31, 2004. The level of seismicity in this stage remains a little higher than that in the Stage 1; however, it is 7.5 times less than the average number of earthquakes occurring during the aftershock sequence (Fig. 3(d)). The distribution of epicenters is random again (Fig. 4(d)) and the  $b$  value returns to about 1 (Fig. 5(d)) as same as that in background stage. The number of days, number of earthquakes and the maximum magnitudes of earthquakes within each stage are listed in Table 1.

### 3. Multifractal spectrum

The temporal and spatial distributions of earthquakes can be considered as fractals with self-similar properties. However, characterizing the complex temporal (or spatial) distributions of earthquakes by using only one fractal dimension may not be comprehensive. The data set with multifractal properties can be considered as an interwoven set with different fractal dimensions (Sprott, 2003). Therefore the multifractal nature of the temporal (or spatial) distributions of earthquakes may implicitly contain several fractal dimensions in the data. The multifractal spectrum is thus a measure of the scaling properties of a data set, which can be characterized by the generalized fractal dimension. The generalized temporal fractal dimension,  $D_q$ , can be



**Fig. 3.** Number of earthquakes per day in the Chiayi area. (a) The first (background) stage from January 1, 1997, to September 20, 1999. (b) The second (foreshock) stage from September 20 to October 22, 1999. (c) The third (aftershock) stage from October 22, 1999, to April 18, 2001. (d) The fourth (background) stage from April 18, 2001, to December 31, 2004.



**Fig. 4.** Spatial distributions of epicenters in the Chiayi area. The red star symbol is the mainshock. (a) The (background) Stage 1. (b) The (foreshock) Stage 2. (c) The (aftershock) Stage 3. (d) The (background) Stage 4.

defined by the following equation (Grassberger and Procaccia, 1983; Hentschel and Procaccia, 1983; Hirata and Imoto, 1991; Kurths and Herzel, 1987):

$$D_q = \lim_{t \rightarrow 0} \frac{\log C_q(t)}{\log t} \quad (1)$$

where  $t$  is the period between the occurrences of two earthquakes, and  $C_q(t)$  is the correlation integral:

$$C_q(t) = \left\{ \frac{1}{N} \sum_{i=1}^N \left[ \left( \frac{1}{N} \sum_{j=1}^N \theta(t - |T_i - T_j|) \right)^{q-1} \right]^{1/(q-1)} \right\}, \quad \text{if } \begin{cases} t \geq |T_i - T_j|, & \theta(x) = 1 \\ t < |T_i - T_j|, & \theta(x) = 0 \end{cases} \quad (2)$$

where  $\theta(x)$  is the Heaviside step function, and  $N$  is the number of earthquakes in the study. In Eq. (1),  $q$  is the real number in the range  $(-\infty, +\infty)$ , and  $T_i$  and  $T_j$  represent the time interval of the  $i$ -th and the  $j$ -th earthquakes' occurrences, respectively. A linear regression of the  $\log(C_q(t))$ - $\log(t)$  relationship for each  $q$  can be used to estimate the generalized fractal dimension. The generalized fractal

dimensions for  $q=0, 1$  and  $2$  are named as the capacity dimension ( $D_0$ ), information dimension ( $D_1$ ) and correlation dimension ( $D_2$ ), respectively.

In regressing the  $D_q$  values, the upper ( $t_s$ ) and lower ( $t_n$ ) boundaries of the scaling interval can avoid the saturation and depopulation of the regression. They can be calculated via the most commonly used correlation dimension ( $D_2$ ) as (Nerenberg and Essex, 1990):

$$t_s = \frac{t}{D_2 + 1} \quad (3)$$

$$t_n = 2t \left( \frac{1}{N} \right)^{1/D_2} \quad (4)$$

Therefore, in each stage we first use all  $C_2(t)$  data to estimate a rough  $D_2$  value. Then, substituting this rough  $D_2$  value into Eqs. (3) and (4) can calculate  $t_s$  and  $t_n$ . Finally, this scaling interval is used to estimate  $D_q$  for  $q = -10$  to  $q = 10$ . An example of the calculations for the background stage (04/18/2001 ~ 12/31/2004) is shown in Fig. 6.

For positive  $q$  values, the  $D_q$  represents the fractal dimension of large fluctuations and the value will be low for multifractal time

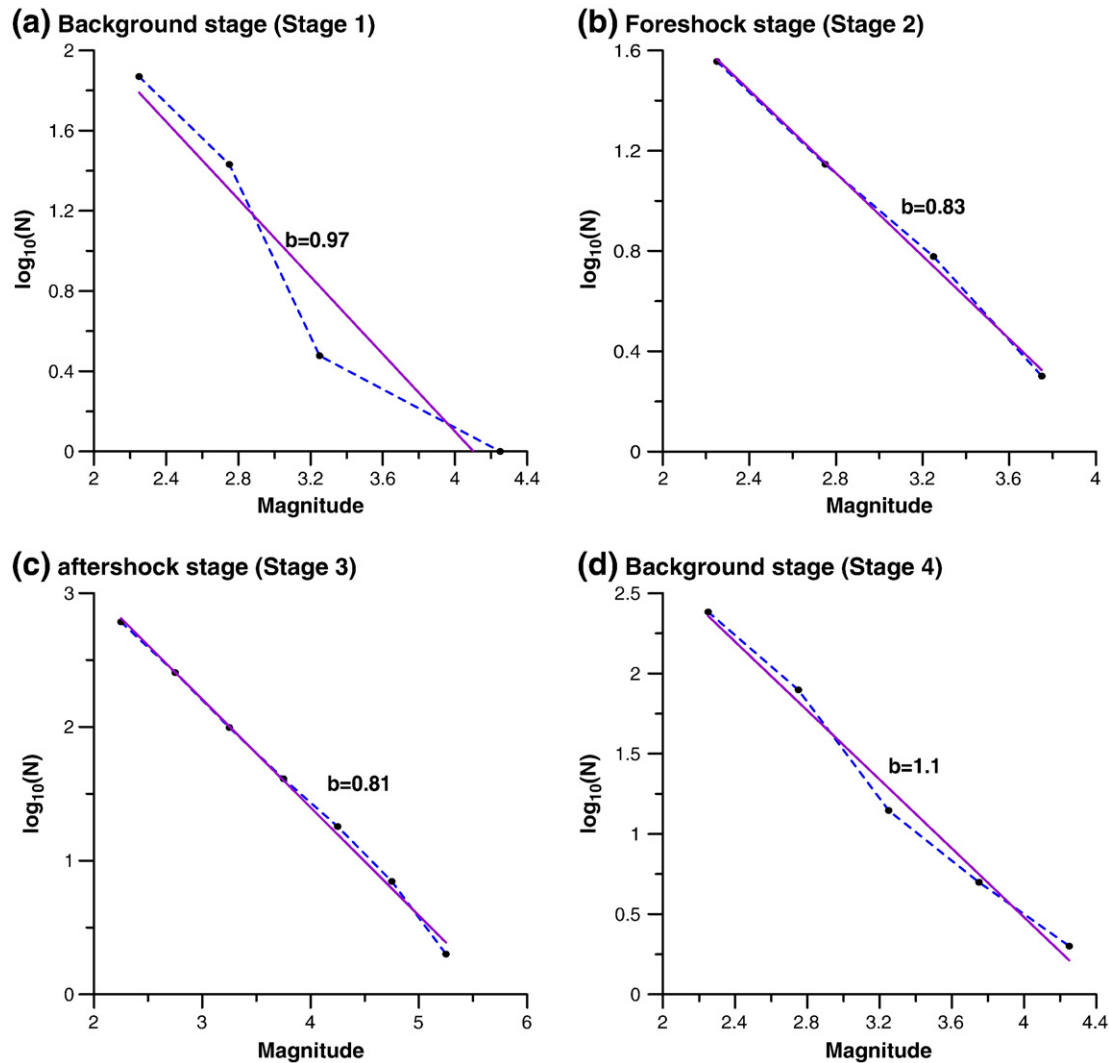


Fig. 5. The  $b$  values of the four stages in the Chiayi area with the magnitude threshold of 2.0. (a) Stage 1. (b) Stage 2. (c) Stage 3. (d) Stage 4.

series. But for negative  $q$  values, the  $D_q$  stands for fractal dimension of small fluctuations and the value will be high for multifractal time series (Sprott, 2003; Telesca et al., 2005). Therefore, in this study, the generalized fractal dimension is helpful for characterizing the temporal seismicities of an earthquake cluster in a small fault area.

#### 4. Results and discussions

To estimate the  $D_q$  of the Chiayi earthquake cluster, the  $q$  values are selected from  $-10$  to  $10$  with a step size of  $2$  for each stage. Estimated  $D_q$  values and their 95% confidence intervals for each stage are shown in Fig. 7. The degree of the multifractality can be represented by the difference between  $D_{-10}$  and  $D_{10}$  and is referred to as the multifractal degree (Telesca et al., 2005). Hence, we interpret the Stage 1 as the background seismicity because its  $D_q$  curve is fairly smooth and its multifractal

degree is about  $0.36$ . The small 95% confidence intervals indicate that the  $D_q$  values of the Stage 1 are reliable. However, the  $D_q$  curve for Stage 2 (the period of enhanced seismicity rate prior to the mainshock) begins to deviate from the Stage 1 response. Instead of gradually dropping as in Stage 1 it retains nearly constant  $D_q$  out to  $q = -2$ . From there, it drops more steeply between  $q = -2$  and  $q = 4$ . For  $q$  higher than  $4$ , Stages 1 and 2 responses coincide. Multifractal degree during Stage 2 remains almost the same as that of Stage 1. Deviation from the Stage 1 response and the steep slope at  $D_0$  are characteristics of Stage 2. In this stage, the 95% confidences are much larger due to the scarcity of data. However, the deviations observed between  $q = -2$  and  $q = 0$  are significant. When the Stage 2 transfers to Stage 3 (aftershock), the  $D_q$  values drop dramatically from  $1.18$  at  $q = -10$  to  $0.24$  at  $q = 10$ , and the multifractal degree increases to  $1.0$ . Nevertheless, the small 95% confidence interval indicates that the results are reliable. Stage 4 marks a return to background seismicity.  $D_q$

Table 1

Numbers of days, earthquakes and maximum magnitude in the four stages of the Chiayi earthquake.

	Background stage (Stage 1)	Foreshock stage (Stage 2)	Aftershock stage (Stage 3)	Background stage (Stage 4)
Period	01/01/97–09/20/99	09/20/99–10/22/99	10/22/99–04/18/01	04/18/01–12/31/04
Days	993	33	545	1354
No. of earthquakes	105	58	1033	342
Maximum magnitude	4.0	3.7	6.4	4.5

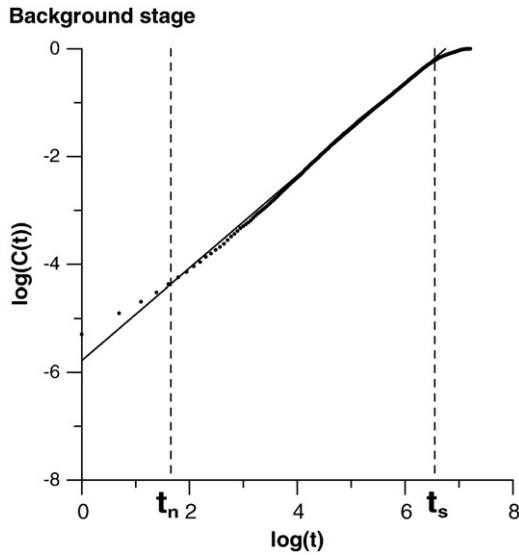


Fig. 6. Plot of the correlation function versus correlation time for  $q = 2$ .  $t_n$  and  $t_s$  are the lower and upper boundaries of the correlation time  $t$ .

remains slightly higher but closely parallels the Stage 1 response. The multifractal degree is the same as that of Stage 1 (0.36). The small confidence interval of this stage indicates that the estimated  $D_q$  values are of the same accuracy as those of the Stage 1. In summary, distinct differences in multifractal behavior are observed in the foreshock and aftershock events associated with the Chiayi earthquake.

In order to test whether the number and threshold magnitude of earthquakes can affect the calculation of the temporal multifractal dimension of the Chiayi earthquake cluster, two tests were performed. In the first test, six test data sets were made and each test data set contains 100 earthquakes which were randomly selected from those of the Stage 3. The threshold magnitudes of earthquakes are  $M_L \geq 0$  for three test data sets and  $M_L \geq 2$  for the other three. The earthquakes in the test data sets are shown in Fig. 8 and their  $D_q$  curves are shown in Fig. 9. For  $q \leq 0$ , the  $D_q$  values indicate the local dimension in the most sparsely populated region (Spratt, 2003) and degree of the small fluctuation of the population (Telesca et al., 2005). For both  $M_L \geq 0$  and  $M_L \geq 2$ , the  $D_q$  curves are steady or descending with the  $q$  value when  $q \leq 0$ . That represents not only the low density of the number of earthquakes in all test data sets but also the homogeneous distribution of

earthquakes within the chosen period. On the contrary, all  $D_q$  curves of test data sets are similar to the original one when  $q \geq 0$  despite that the earthquake numbers of test data sets are significantly smaller than that in the Stage 3. For the second test, the  $D_q$  values in the foreshock stage are calculated for earthquakes with magnitudes  $M_L \geq 0$  and  $M_L \geq 2$  (Fig. 10), and the numbers of earthquakes in the data sets are 110 and 58, respectively. Fig. 10 shows that their multifractal properties are alike. Therefore, based on the two tests we can conclude that the number and threshold magnitude of earthquakes do not influence the calculation of the multifractal property significantly.

The large fluctuation of  $D_q$  of the spatial distribution of earthquakes in the range  $D_2 \sim D_\infty$  generally means an increase in the heterogeneity of the earthquakes distributions (Lei et al., 1993). The same idea can be used to the  $D_q$  of the temporal distribution of earthquakes. In the Stage 1, the  $D_q$  curve is smooth with negligible fluctuation. Therefore, the earthquake cluster in the background stage is considered to be nearly homogeneous (or nearly monofractal) in the temporal distribution of earthquakes. Nearly monofractal dimension means that the fractal dimension is insensitive with  $q$ . For the Stage 2, the earthquake cluster has evolved to be heterogeneous and multifractal as the  $D_q$  curve is more fluctuant than that in the background stage. Following the same line, the earthquake cluster in the Stage 3 should be the most heterogeneous and strongly multifractal with the largest multifractal degree because the fluctuation of the  $D_q$  in this stage is the largest. Finally, characteristics of the earthquake cluster in the Stage 4 should be similar to that in the Stage 1 because both of which show approximately the same pattern of the  $D_q$  curve and the same multifractal degree. Therefore the temporal multifractal property of seismicity becomes noticeable when the seismicity becomes active, and vice versa.

In addition, we used the moving window method to calculate the  $D_q$  curves of the Chiayi earthquakes with  $M \geq 2$  (Fig. 11). The window length is 33 days except the window A which is from Jan. 1, 1997 to Sep. 20, 1999 because of too few earthquakes at background stage within 33 days window length to calculate the  $D_q$  curve. The  $D_q$  curve of the window A is smooth. When the window is moved to B position which straddles the background and foreshock stages, the  $D_q$  curve becomes more curvy than those of window A especially with a significant slope change around the  $q = 0$ . For the window C which is within the foreshock stage, the  $D_q$  curve returns and is similar to those of window A. Therefore using the moving window to scan the temporal relationships of earthquakes of an area, the change of slope of  $D_q$  curve at  $q = 0$  may be used as an indicator for the foreshock. A similar variation is observed from windows C to E but more violent. Window D straddles the foreshock and aftershock stages, the  $D_q$  curve is curviest and the fractal degree is largest among all windows. Window E is within the aftershock stage, the fractal degree is smaller than that of window D but it is still larger than others. From windows F to G, their  $D_q$  curves are becoming smooth gradually.

According to the results of the moving window analysis, if the window straddles two different stages (two different populations of data), the  $D_q$  values will vary significantly since this method enhances the contrast between the short and long temporal correlations of earthquakes. As a result, a large fluctuation of  $D_q$  is produced. This will lead to a misinterpretation that the study area is under a strong heterogeneous state. For our study area, however, each stage has its own seismogenic characteristic and seismicity. Therefore, multifractal characteristics of a stage can only be revealed by that stage's seismic data. According to Rossi (1990), the fractal dimension in the spatial distributions of hypocenters ( $D_F$ ) decreases before and after a large earthquake. Nevertheless, our results (Figs. 7 and 11) show that  $D_q$  for  $q \geq 2$  also decreases before and even after a large earthquake but the  $D_q$  increases for  $q \leq -2$ . Additionally,  $D_q$  values for  $q = 0$  are almost the same in all stages. This means that  $D_0$  is relatively insensitive to the seismicity in time. This result is consistent with the conclusion of Wang and Lee (1996) that  $D_0$  is not able to distinguish spatial fractal properties of earthquakes.

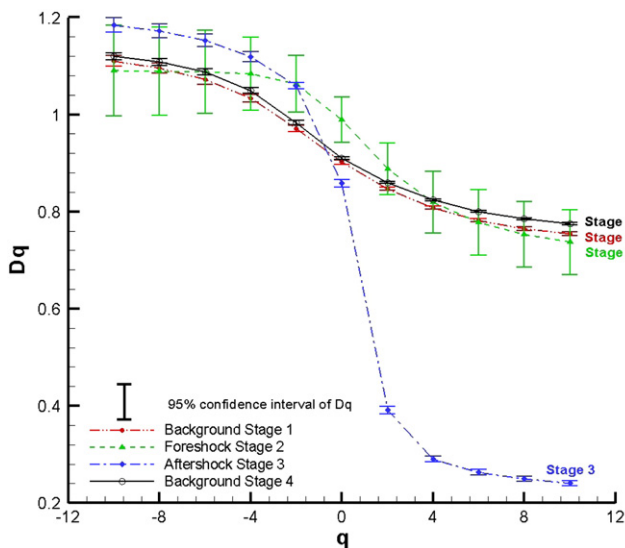
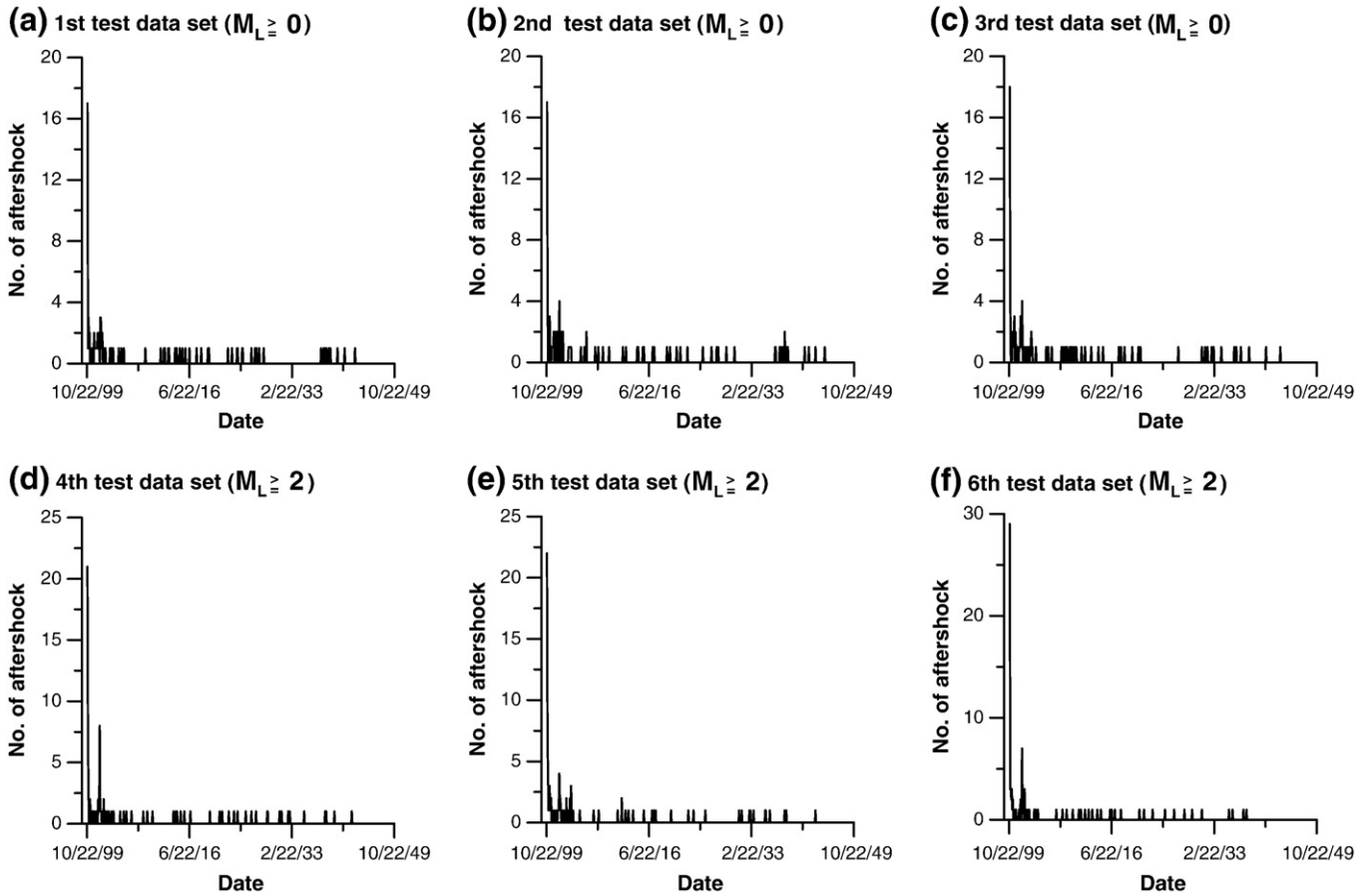


Fig. 7. Fluctuations of the generalized fractal dimensions  $D_q$  ( $q = -10$  to  $q = 10$ ) and their 95% confidence intervals for the four stages of the Chiayi earthquake.



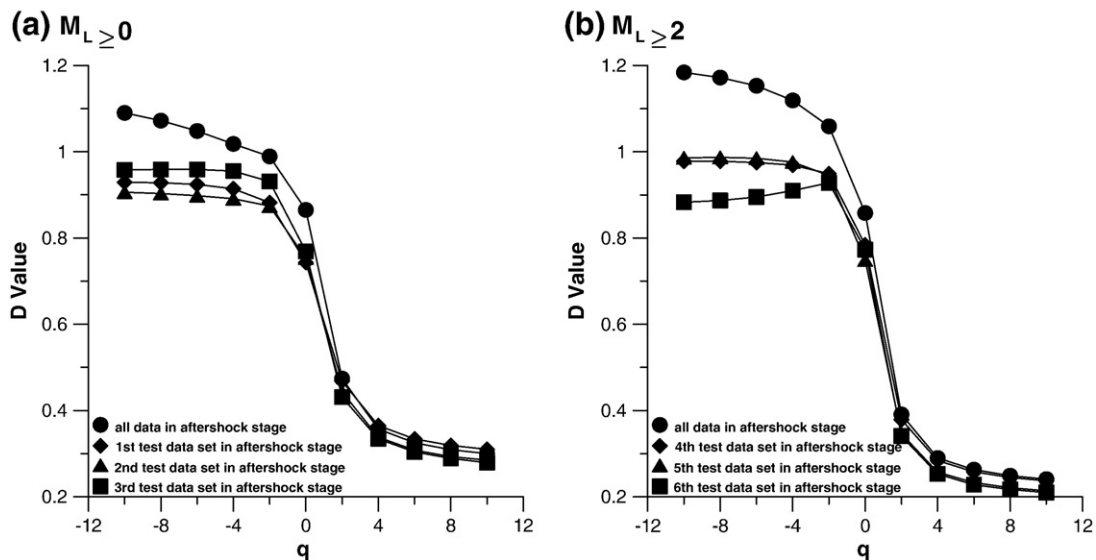
**Fig. 8.** Earthquake sequences of six test data sets and each containing 100 earthquakes which were randomly selected from the aftershock stage with  $M_L \geq 0$  (a) to (c) and  $M_L \geq 2$  (d) to (f).

Below we summarize the temporal multifractal properties of the four stages identified in retrospect from the Chiayi earthquake:

**Background Stage (Stage 1):** The fluctuation of  $D_q$  from  $q = -10$  to 10 for the background seismicity in the Chiayi area is smaller than those of other stages with the multifractal degree of  $D_q$  being less than 0.36. The average value of  $D_q$  in this stage is about 1. This nearly monofractal property

may mean that the background seismicity is generated by a stochastic process (Godano and Caruso, 1995; Godano et al., 1999). Hence the background seismicity of the Chiayi earthquake can be regarded as a result of nearly monofractal stochastic process.

**Foreshock Stage (Stage 2):** The seismicity during the foreshock stage increases in frequency.  $D_q$  drops more steeply from negative to positive  $q$  (Fig. 7). In the foreshock



**Fig. 9.** The D values of six test data sets and each containing 100 earthquakes which were randomly selected from the aftershock stage with (a)  $M_L \geq 0$  and (b)  $M_L \geq 2$ .

## Foreshock stage

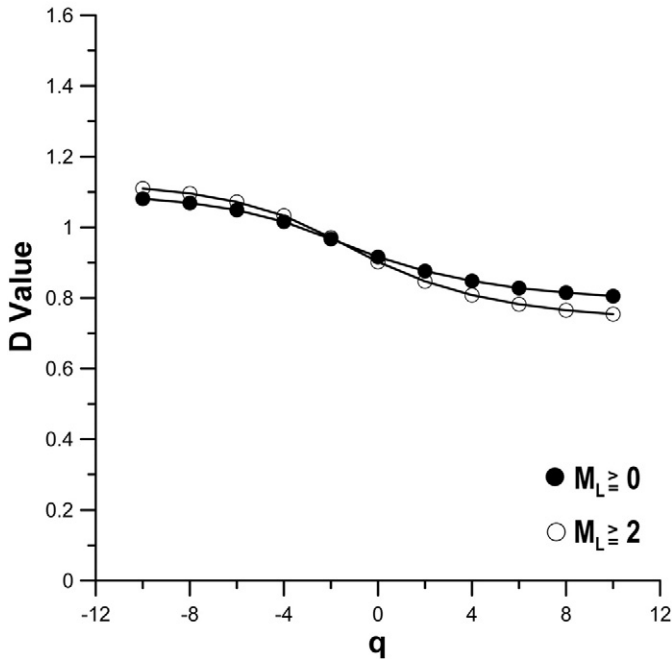


Fig. 10. The  $D$  values of the foreshock stage with different threshold magnitude ( $M_L \geq 0$  and  $M_L \geq 2$ ).

stage, the spatial fractal dimension decreases with time because of the active seismicity before a large earthquake (Godano et al., 1999; Kiyashchenko et al., 2003; Nakaya and Hashimoto, 2002). However for the Chiayi earthquakes, the  $D_q$  is higher than the background Stage 1 data roughly between  $q$  equal  $-2$  to  $0$ . In addition, a drop in  $b$  value is also observed in this stage which is consistent with the conclusions of Suyehiro (1966), Henderson et al. (1992) and Smith (1998). It is

known that the variation of the multifractal properties of earthquakes is caused by the changes in the seismogenic regime (Mittag, 2003). Furthermore, the static stress changes in the Chiayi area, which is simulated by the observed creep rate variations with regard to the velocity-strengthening friction law and 1-D groundwater diffusion model, were transferred from the Chichi earthquake (Chang and Wang, 2006). In addition, the anomalous diurnal variations of geomagnetic field and resistivity before the Chiayi earthquake were observed (Liu et al., 2001, 2006; Tsai et al., 2006; Yen et al., 2004). The foreshock period is characterized not only by anomalous seismic behavior, but is accompanied by anomalous geomagnetic and electrical behaviors as well.

**Aftershock Stage (Stage 3):** In the aftershock stage,  $D_0$  is close to  $0.86$  but  $D_{10}$  drops to  $0.24$ , the lowest value of  $D_q$ . The difference between  $D_0$  and  $D_{10}$  of this stage is the greatest in all of the four stages, and the multifractal degree is also the largest one ( $1.0$ ). Therefore, the temporal multifractal property of the aftershock stage is clearly observed, and the heterogeneity of the seismicity in the aftershock stage is the strongest in all of the four stages.

Aftershock is a procedure of releasing energy (Utsu, 1961). The released energy reduces with time, and the numbers of the aftershocks also decrease over time (Omori, 1894). Furthermore, there are two diffusive aftershock mechanisms (Telesca et al., 2005): 1) Mainshock does not release the stress completely. 2) The stress field is modified after mainshock. These theories imply that energy accumulation and energy release processes co-exist in aftershocks. Therefore, the seismogenic mechanics in the aftershock stage is under a stress-adjusting state. Accordingly, the seismogenic regime in the Chiayi area becomes more complex and active. This complex seismogenic system yields the significant temporal aftershock distributions and the complex temporal multifractal properties.

**Background Stage (Stage 4):** After the aftershock stage, the seismicity in the Chiayi region returns to the background seismicity

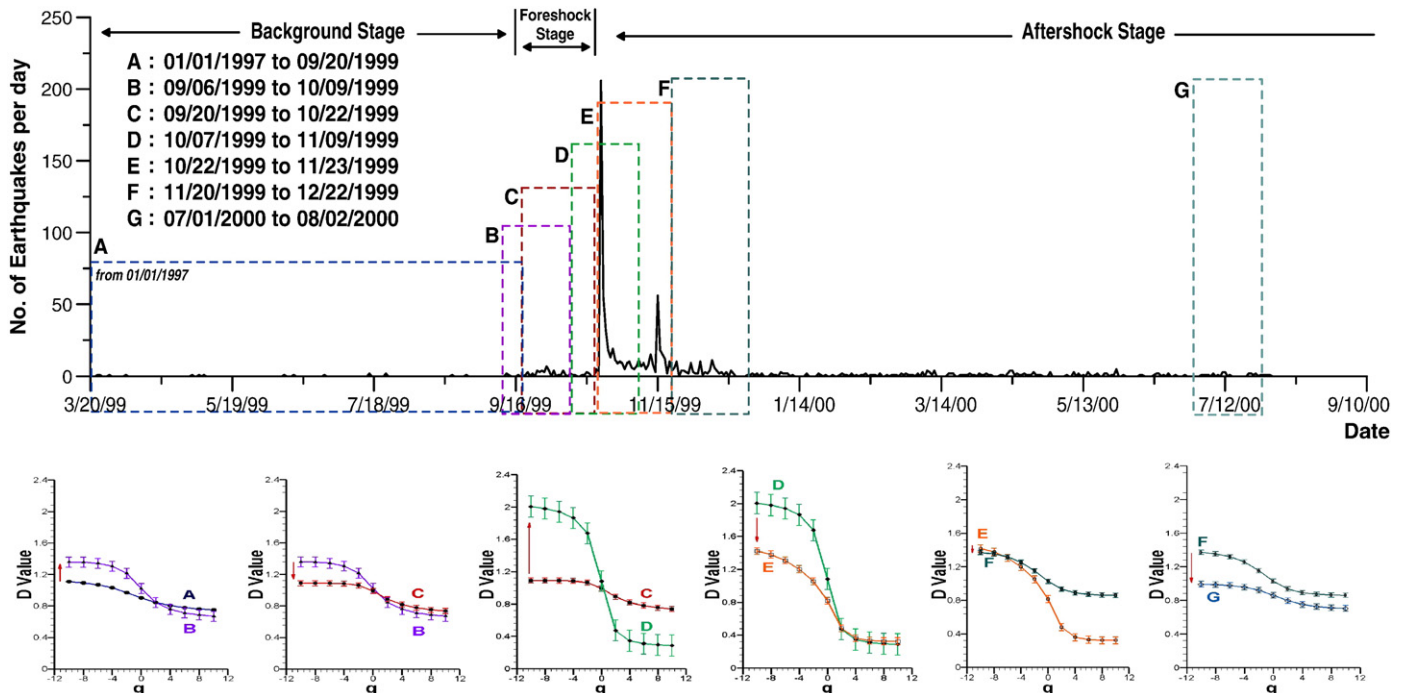


Fig. 11. The  $D_q$  values of the seismicities within the windows are calculated at 7 different time positions (A–G) from the background, foreshock stages and to the aftershock stage.

with the same multifractal degree as that in the Stage 1. Therefore, the temporal multifractal spectrum of seismicity in the aftershock stage becomes more stabile.

## 5. Conclusions

The fluctuations in the temporal multifractal dimension of the Chiayi earthquake cluster reveal significant differences between temporal seismogenic regimes. For the background seismicity, the  $D_q$  drops sinusoidally from about 1.12 to 0.8 as  $q$  increases from  $-10$  to  $10$ . A distinct foreshock stage is observed in the multifractal curve (Stage 2). It deviates from the background response and remains higher for  $q$  between  $-4$  and  $4$ . These differences are statistically significant between  $q$  of  $-2$  and  $2$ . In the aftershock stage (Stage 3),  $D_q$  drops from about 1.1 to 0.4 as  $q$  increases from  $-2$  to  $2$  which is much larger than those in the other stages. Over the entire range,  $D_q$  has multifractal degree of about 1 (dropping from about 1.2 to 0.24). Finally, the seismicity returns to background levels (Stage 4) and the  $D_q$  curve takes on the smoother sinusoidal variations characteristic of Stage 1.

This study illustrates that the variations in the temporal generalized fractal dimension of an earthquake sequence can reveal statistically significant differences between the various stages associated with background, foreshock and aftershock seismic behavior. The statistically significant changes of  $D_q$  curve are observed in the slope at  $D_0$  and suggest these measures may be used to identify precursor activity.

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