

Probability-based PGA estimations using the double-lognormal distribution: Including site-specific seismic hazard analysis for four sites in Taiwan

J.P. Wang^{a,*}, Logan Brant^b, Yih-Min Wu^c, Hamed Taheri^a

^a Dept. Civil and Environmental Engineering, the Hong University of Science and Technology, Kowloon, Hong Kong

^b Dept. Civil Engineering and Engineering Mechanics, Columbia University, New York City, USA

^c Dept. Geosciences, National Taiwan University, Taipei, Taiwan

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ABSTRACT

This study develops a probabilistic approach for examining site-specific seismic hazards using data from 438 $M_w \geq 5.5$ earthquakes and attenuation relationships local to the region around Taiwan. These are combined to generate semi-observed peak ground accelerations, in a form which can be modeled by a double-lognormal distribution. This model, which satisfies statistical goodness-of-fit, provides the relationship between the exceedance probability and a given ground motion level. The study includes site-specific seismic hazard analysis for four nuclear power plant sites in Taiwan. The results show that the seismic hazards at the four sites are not the same. While no seismic hazard analysis is without challenge, a troublesome trend appears that many applications of decision making are being influenced by the complexity of the calculation, instead of how well the fundamentals of the analysis are understood and can be verified. The study provides an analysis which is not overly complicated, is in good agreement with an empirical control, and offers transparency, traceability, and verifiability.

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1. Introduction

The region around Taiwan is known for high seismic risk owing to its unique tectonic setting. Catastrophic earthquakes, such as the Chi-Chi earthquake in 1999, have pounded the island and caused extensive damage. In preparing a plan for earthquake hazard mitigation, the question which needs to be answered is not “whether” future catastrophic events will occur, but rather “when”, “where” and “how large” the event will be. Wang et al. [1] studied the distribution of annual maximum earthquakes since the year 1900 and suggested a five percent probability for a catastrophic earthquake event striking Central Taiwan within the next 50 years.

Geoscientists and civil engineers have spent countless time and effort studying earthquake engineering and hazard mitigation for the study region. Taiwan’s Central Geological Survey, have been investigating active faults in Taiwan and publishing these results periodically [2,3] and several earthquake early warning systems have been developed specifically for the region [4–7]. Despite all the recent improvements, the probabilistic seismic hazard analysis (PSHA) method proposed in the late 1960s [8] remains the most important tool connecting to performance-based earthquake resistant design [9,10]. It has evolved into the customary approach used for seismic hazard assessment of

nuclear power plant sites [11]. However, methodological limitations of PSHA have been criticized [12–14]. One issue involves its use of the logic tree, requiring a number of subjective judgments. Krinitsky argues that the allocation of these numbers is meaningless from a probabilistic perspective [15]. This weakness is demonstrated in a recent PSHA study for Taiwan [16], which made use of a large and very complex logic tree containing hundreds of branches, but without offering support for how the selection of those numbers was made. The results of that study are discussed again later in this paper.

The deterministic seismic hazard assessment method, an alternative to PSHA, is an approach with its own limitations [15]. As Mualchin has indicated [17], no seismic hazard approach is perfect. Before the method is applied for seismic hazard analysis the fundamentals of the problem must be clearly understood by the decision makers involved [17]. Increased complexity does not necessarily improve the accountability of results, given current limitations in the knowledge of the earthquake process [17]. Reliable hazard estimates should be developed from a logical, transparent, and verifiable framework [17,18].

This study develops a statistical approach for examining site-specific seismic hazards using data from 438 $M_w \geq 5.5$ earthquakes since 1900 and attenuation relationships local to the region around Taiwan. These are combined to generate semi-observed peak ground accelerations, in a form which might be modeled by a double-lognormal distribution. This model, which satisfies a statistical goodness-of-fit, provides the relationship between the exceedance probability and a given ground motion

* Corresponding author.

E-mail address: jpwang@ust.hk (J.P. Wang).

level. The procedure is demonstrated during the evaluation of the respective seismic hazards at four nuclear power plant (NPP) sites in Taiwan.

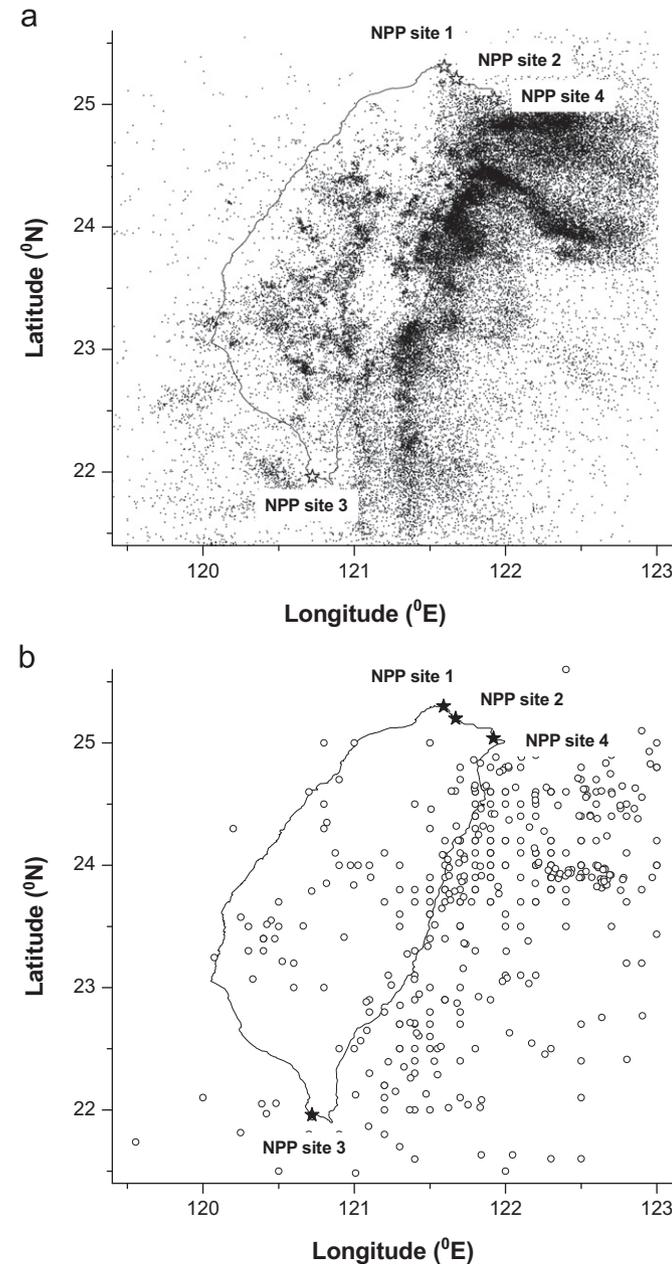


Fig. 1. Spatial distribution of the declustered earthquakes from the catalog since 1900 around Taiwan: (a) a total of more than 57,000 earthquakes regardless of size, and (b) $M_w \geq 5.5$ events.

2. Semi-observed peak ground acceleration

The PGA estimated from using actual seismicity and local ground motion models is referred to as semi-observed peak ground acceleration (SOPGA) in this paper. Note that it is not the direct PGA measurement during earthquake occurrences. To sample the most SOPGA data possible, this study uses an extensive earthquake catalog. After declustering, this catalog includes more than 57,000 events around the study region. The records go back as far as 1900, with some incompleteness in the catalog before 1972 for small magnitudes ($3.0 \leq M < 4.0$) and 1933 for moderate magnitudes ($4.0 \leq M < 5.0$), respectively [1]. Fig. 1(a) shows the spatial distribution of the earthquakes contained within the catalog. Only the most intense earthquakes (i.e., $M_w \geq 5.5$) were used for the SOPGA calculations. Fig. 1(b) shows the distribution of the catalog's 438 largest events.

The study utilized only ground motion attenuation models previously used for seismic hazard analysis in Taiwan [16]. Table 1 summarizes the expressions along with their model standard deviations ($\sigma_{\ln Y}$). The four ground motion models selected for this study were each assigned equal weight for computing the SOPGA at the four NPP sites, since specific site characteristics were not well established for these locations.

3. Mean and mean+SD SOPGA

The general equation of ground motion models is presented as:

$$\ln Y = f(M, D); \quad \sigma_{\ln Y} = \sigma^* \tag{1}$$

where f denotes the prediction function; M and D are magnitude (in moment magnitude, local magnitude, etc.) and source-to-site distance, respectively; Y is ground motion and it represents PGA specifically in this study; σ^* (a constant) is the model standard deviation (SD). $f(M, D)$ is the mean of $\ln Y$. Through Eq. (1), the mean and mean+SD motions (denoted as Y_m and Y_{m+sd} , respectively) become:

$$Y_m = \exp(f(M, D)) \tag{2}$$

$$Y_{m+sd} = \exp(f(M, D) + \sigma_{\ln Y}) \tag{3}$$

The mean+SD SOPGA motion conservatively accounts for the uncertainty in ground motion models, while the less conservative mean SOPGA motion provides a central value that does not consider such variability.

4. Magnitude and distance thresholds

For this study, only moderate to large earthquakes within a certain distance from the study site are of significance, since only these events have SOPGA large enough to potentially damage engineered structures. A magnitude threshold (m_0) of 5.5 (in M_w)

Table 1
Ground motion models used in a recent seismic hazard study in Taiwan (after Cheng et al. [16]).

Description	PGA attenuation relationship	$\sigma_{\ln Y}$
Hanging wall, rock, crustal	$\ln y = -3.25 + 1.075M_w - 1.723 \ln(D + 0.156 \exp(0.624M_w))$	0.577
Hanging wall, soil, crustal	$\ln y = -2.80 + 0.955M_w - 1.583 \ln(D + 0.176 \exp(0.603M_w))$	0.555
Foot wall, rock, crustal	$\ln y = -3.05 + 1.085M_w - 1.773 \ln(D + 0.216 \exp(0.612M_w))$	0.583
Foot wall, soil, crustal	$\ln y = -2.85 + 0.975M_w - 1.593 \ln(D + 0.206 \exp(0.612M_w))$	0.554

Note: M_w =moment magnitude; D =source-to-site distance in km; y =PGA in g

was selected for this study. A distance threshold (d_0) of 200 km was also applied.

After applying the magnitude and distance threshold values to the data from the earthquake catalog, the numbers of $M_w \geq 5.5$ earthquakes in the vicinity of the four NPP sites reduce to 280, 290, 145, and 301 earthquakes, respectively, for the NPP sites 1 through 4. This corresponds to annual occurrence rates of 2.55, 2.64, 1.32, and 2.74, respectively. NPP sites 1, 2 and 4 are located in the northern part of Taiwan and NPP site 3 is located in the south, as shown on Fig. 1. Note the high concentration of earthquake occurrences observed within 200 km of NPP site 4.

The thresholds used in this study were based on engineering judgment. Similar assumptions are needed in other types of seismic hazard analyses. To the best of the authors' professional experience and knowledge, a distance threshold of 200 km is commonly used when conducting PSHA for NPP developments.

5. Double-lognormal distribution and statistical goodness-of-fit testing

To demonstrate the analytical procedure involved with the statistical goodness-of-fit evaluation we consider NPP site 4 as an example. Fig. 2(a) shows the spatial distributions of the 301 $M_w \geq 5.5$ earthquakes within 200 km from this site. Fig. 2(b) shows the mean SOPGA calculated using Eq. (2). The data forms an asymmetrical distribution, which does not allow it to be simulated with a symmetrical probability model, such as a normal distribution. To select the “best-estimate” model, either trial-and-error or variable

transformation techniques can be performed. The former compares the relative appropriateness among different models, and the latter compares the relative appropriateness among different transformed variables using the same model. This study used variable transformation to search for a model that is statistically satisfied. Fig. 2(c) and (d) show the single logarithm and double logarithm of mean SOPGA, denoted as $\ln(\text{SOPGA}_m)$ and $\ln(\ln(\text{SOPGA}_m))$, respectively. The level of symmetry of the transformed data is higher using a double logarithm compared to a single logarithm. Owing to this symmetry, the series of the transformed data in the double-lognormal form could be more likely modeled by a normal distribution.

Statistical goodness-of-fit testing was used in this study to evaluate this selection. Specifically, the Kolmogorov–Smirnov (K–S) test was adopted at a level of significance (α) equal to five percent, common for statistical testing of this type [1]. For the K–S test the selected probability model is not rejected by statistical evidence when the maximum difference between the observed and theoretical cumulative probabilities less than the critical value [1,20]. The theoretical probability can be determined using conventional probability methods and the observed probability in the K–S test is obtained by the following expression [1,20]:

$$S_n(x) = \begin{cases} 0 & x < x_1 \\ k/n & x_k \leq x < x_{k+1} \\ 1 & x \geq x_n \end{cases} \quad (4)$$

where n is the number of observation and x_k denotes the k -th observation in an ascending order. Fig. 3 compares the observed and theoretical cumulative probabilities for the two series of

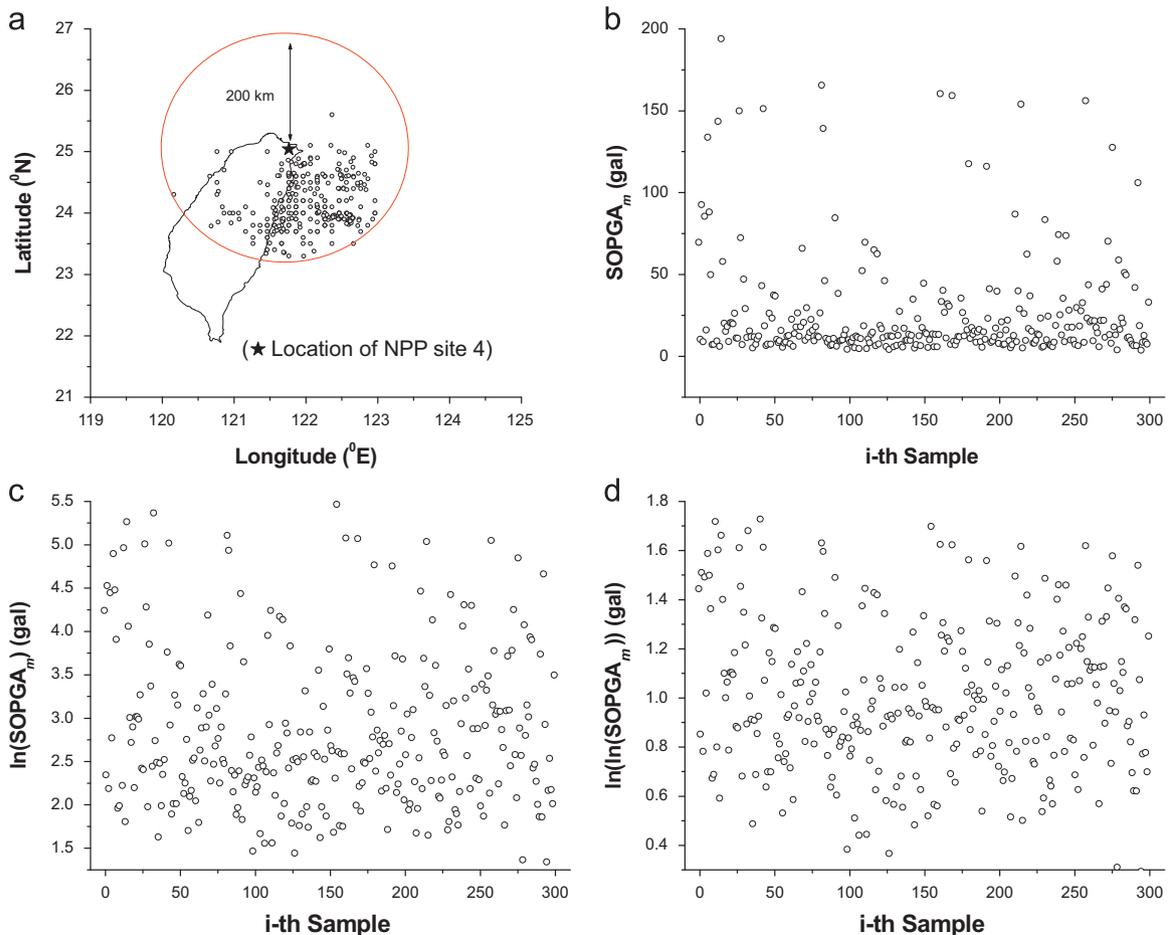


Fig. 2. (a) Spatial distribution of 301 $M_w \geq 5.5$ earthquakes 200 km within NPP site 4, (b) the distribution of 301 mean SOPGAs (denoted as SOPGA_m), (c) the distribution of 301 $\ln(\text{SOPGA}_m)$, and (d) the distribution of 301 $\ln(\ln(\text{SOPGA}_m))$.

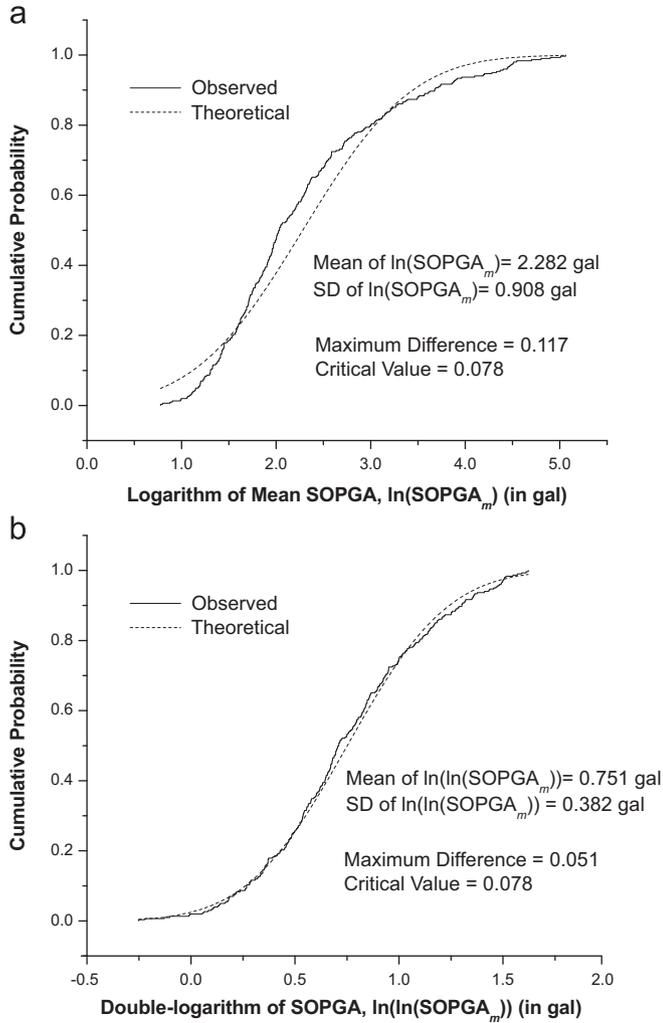


Fig. 3. Results of K-S goodness-of-fit tests: (a) 301 $\ln(\text{SOPGA}_m)$ simulated by a normal distribution, and (b) 301 $\ln(\ln(\text{SOPGA}_m))$ simulated by a normal distribution.

transformed data. With 301 samples under a five percent significance level, the critical value is 0.078, and the maximum differences are 0.117 and 0.051 for $\ln(\text{SOPGA}_m)$ and $\ln(\ln(\text{SOPGA}_m))$, respectively. The test suggests that the $\ln(\ln(\text{SOPGA}_m))$ series might be modeled by a normal distribution, but not the $\ln(\text{SOPGA}_m)$ series. Fig. 4 shows the Q-Q plots for the two types of data compared to the normal distribution. The level of fitting is also better for $\ln(\ln(\text{SOPGA}_m))$ than for $\ln(\text{SOPGA}_m)$. In other words, this specific series of mean SOPGA for NPP site 4 might be modeled by a double-lognormal distribution from both statistical evidences. On the other hand, since the SOPGA is modeled by a double-lognormal distribution, the data must contain only non-negative values.

The procedures were repeated for the three other sites and for the other motion, mean+SD SOPGA. Table 2 summarizes the statistics (e.g., mean value, standard deviation, etc.) and the results of K-S tests. The double-lognormal distribution satisfied the statistical goodness-of-fit for each of the eight examples evaluated.

6. PGA seismic hazard curve

The model can be utilized to estimate the relationship between annual exceedance rate and ground motion level, which is usually referred to as a seismic hazard curve [19]. Using the double-

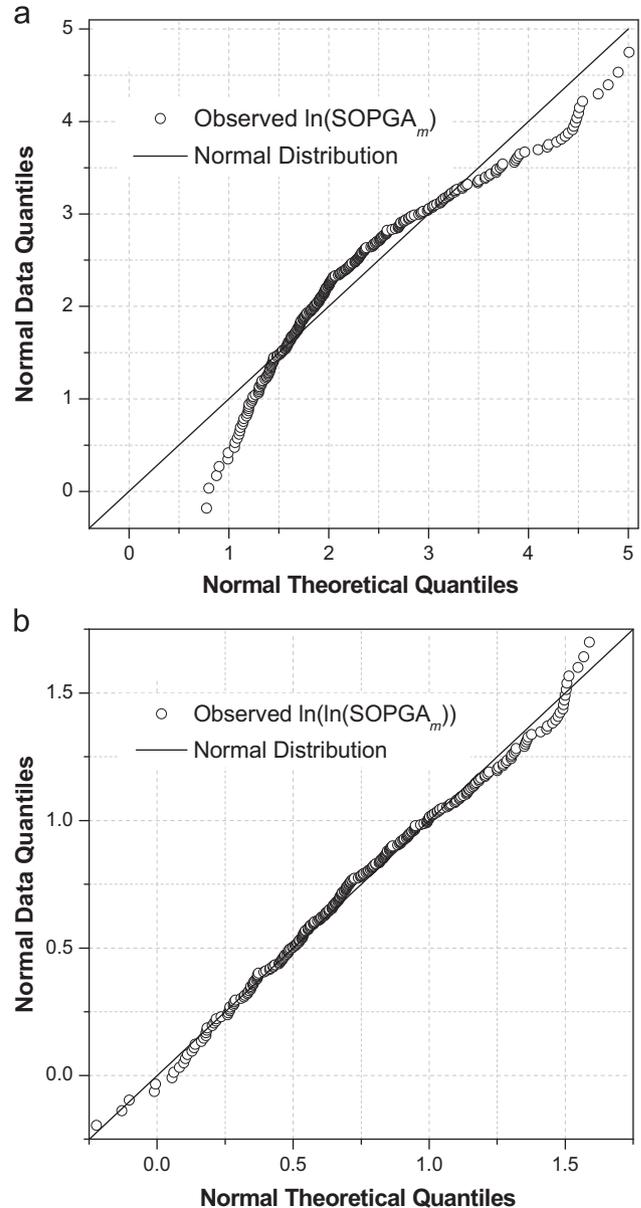


Fig. 4. Statistical goodness-of-fit presented in Q-Q plots: (a) 301 $\ln(\text{SOPGA}_m)$ compared to a normal distribution, and (b) 301 $\ln(\ln(\text{SOPGA}_m))$ compared to a normal distribution.

lognormal distribution, the probability of Y exceeding a given motion, denoted as $\Pr(Y > y^*)$, can be derived as follows [20]:

$$\Pr(Y > y^* | \mu_d, \sigma_d) = 1 - \Pr(Y \leq y^* | \mu_d, \sigma_d) = 1 - \Phi\left(\frac{\ln(\ln(y^*)) - \mu_d}{\sigma_d}\right) \quad (5)$$

where μ_d and σ_d are the mean value and standard deviation of $\ln(\ln(\text{SOPGA}))$, respectively; Φ is the standard normal cumulative function. Following the framework of the PSHA [19], for a given mean earthquake rate (ν) the exceedance rate (λ) for a certain period of time becomes:

$$\begin{aligned} \lambda(y^*, \nu, \mu_d, \sigma_d) &= \nu \times \Pr(Y > y^* | \mu_d, \sigma_d) \\ &= \nu \times \left(1 - \Phi\left(\frac{\ln(\ln(y^*)) - \mu_d}{\sigma_d}\right)\right) \end{aligned} \quad (6)$$

Note that the seismic hazard estimated by Eq. (6) is rate-based, not probability-based. This is because the hazard can exceed 100% (for instance if ν is very large), violating a fundamental rule of probability.

Table 2

Statistics of the mean and mean+SD for $\ln(\ln(\text{SOPGA}))$ back-calculated from respective series of $M_w \geq 5.5$ earthquakes within 200 km from four nuclear power plants in Taiwan; the results of K-S tests on the series of motions compared to a normal distribution are also summarized.

Variable	Statistics/K-S results	Site 1	Site 2	Site 3	Site 4
$\ln(\ln(\text{SOPGA}_m))$	Sample size	280	290	145	301
	Mean value (gal)	0.550	0.618	0.693	0.751
	Standard deviation (gal)	0.392	0.383	0.431	0.382
	Annual rate (1 year)	2.545	2.636	1.318	2.736
	Maximum difference	0.042	0.039	0.054	0.051
	Critical values	0.081	0.080	0.113	0.078
$\ln(\ln(\text{SOPGA}_{m+sd}))$	Goodness-of-fit	Yes	Yes	Yes	Yes
	Sample size	280	290	145	301
	Mean value (gal)	0.845	0.896	0.957	0.999
	Standard deviation (gal)	0.297	0.295	0.333	0.302
	Annual rate (1 year)	2.545	2.636	1.318	2.736
	Maximum difference	0.060	0.056	0.064	0.067
Critical values	0.081	0.080	0.113	0.078	
Goodness-of-fit	Yes	Yes	Yes	Yes	

Fig. 5 shows the respective rate-based annual hazard curves represented by Eq. (6). The results show that NPP sites 3 and 4 are subject to higher seismic hazard compared to NPP sites 1 and 2. Fig. 6 illustrates why $\ln(\ln(\text{SOPGA}_{m+sd}))$ with higher mean values and lower standard deviations (the narrower distribution) presents a smaller right-tail probability, $\Pr(Y > y^*)$, when y^* is large. As a result, the wider distribution of the mean motion can have a larger right-tail probability for the same y^* , despite its lower central value.

7. Probability-based hazard curves

The aforementioned hazard curves describe the relationship between exceedance rate and PGA, which is not a probability-based relationship. For developing probability-based seismic hazard, a Poisson distribution was used following the computation in PSHA [19]. Given an annual rate of exceedance (λ), the exceedance probability for y^* within a given period of time (t) is as follows [19]:

$$\Pr(Y > y^* | \lambda, t) = 1 - e^{-\lambda \times t} \tag{7}$$

where λ at y^* can be obtained through Eq. (6). Fig. 7 shows the annual probability-based distribution ($t=1$) generated using Eq. (7). The results show that annual exceedance probabilities are close to their counterparts (annual exceedance rate).

8. Discussions

8.1. Recommended seismic hazard

A variety of seismic hazards have been developed in this study for specific analytical conditions (e.g., assumptions, limitations, etc...), such as mean or mean+SD motions, threshold values, and the rate-based or probability-based hazard. We recommend that the probability-based seismic hazard utilizing the mean+SD SOPGA be used, for developing earthquake resistant designs at the four NPP sites considered in this study. This approach accounts for the uncertain nature of the ground motion attenuation. The probability-based hazard is beneficial for performing risk analysis incorporating failure consequences. For a design PGA equal to 0.5g, the annual exceedance probabilities are close to 0.1%, 0.2%, 0.6%, and 0.9% at NPP sites 1 through 4, respectively. Such seismic hazards are logically and statistically sound. The variations in these estimates are influenced by the occurrence

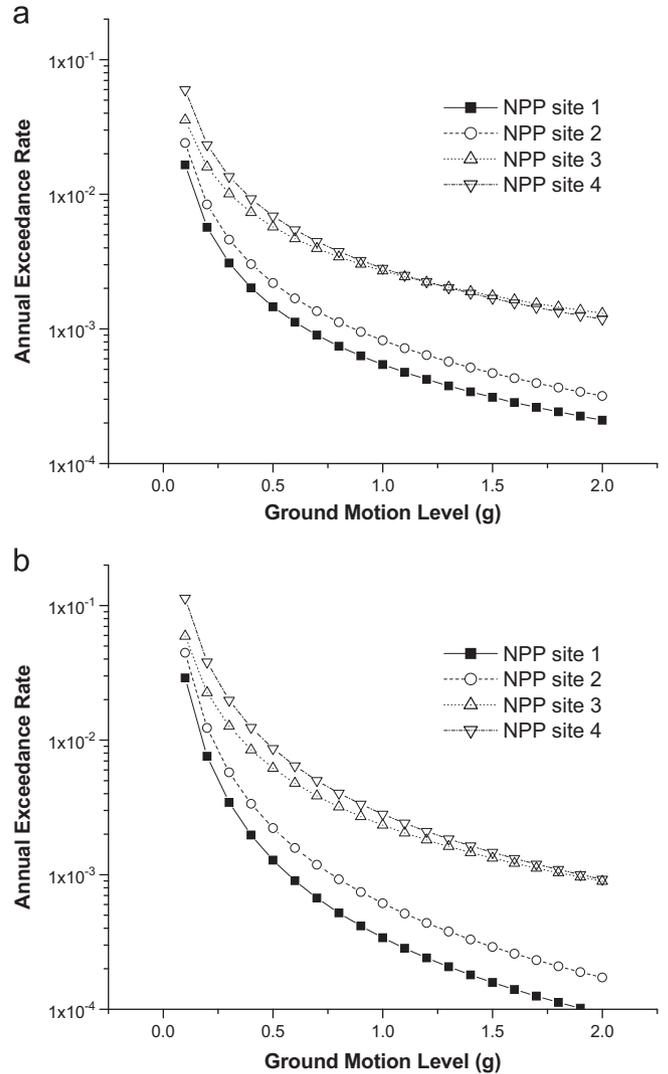


Fig. 5. Rate-based seismic hazard curves for the four sites: (a) based on mean SOPGA, and (b) based on mean+SD SOPGA.

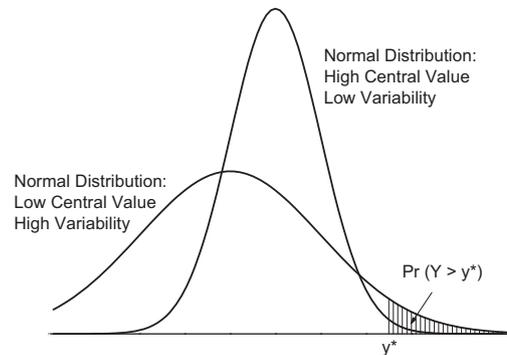


Fig. 6. Systematic diagram for different normal probability distributions and their right-tail probability (exceedance probability).

rate of the defined damaging earthquakes ($M_w \geq 5.5$ events within 200 km) and their uncertain size and location.

8.2. Methodology robustness and result accountability

Klugel [18] conducted a comprehensive review of seismic hazard analysis, in which he commented on some characteristics

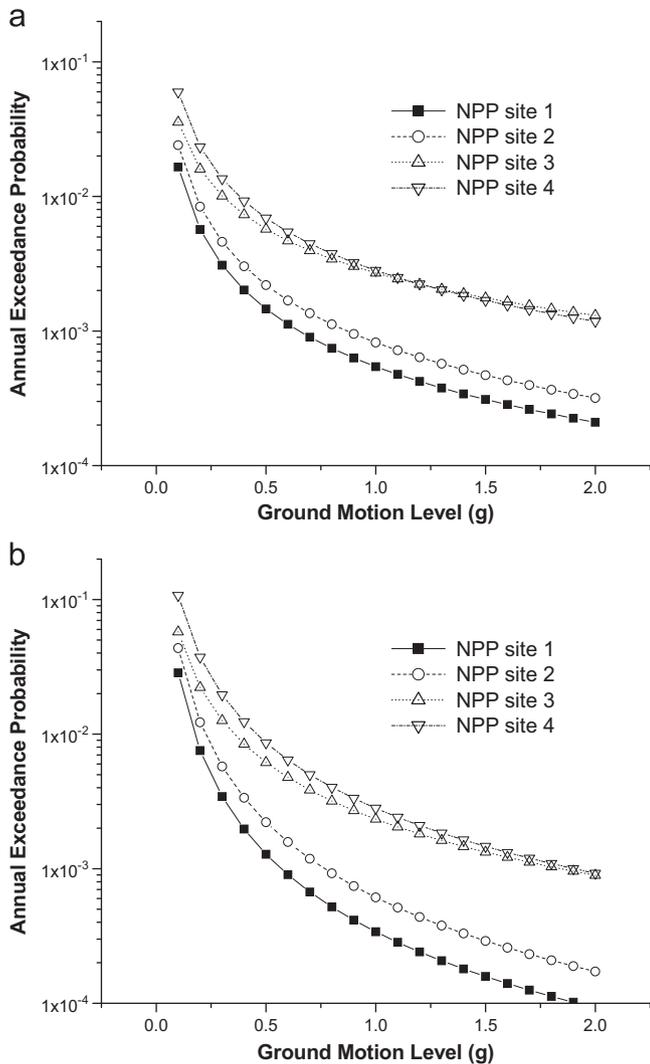


Fig. 7. Relationship between annual exceedance probability and ground motion level for the four sites: (a) based on mean SOPGA, and (b) based on mean+SD SOPGA.

of a robust analysis, including traceability and transparency. From Klugel's perspective, transparency requires that key assumptions be clearly documented and understood and traceability requires that the result can be repeated and verified. Applying this standard to both this study and the recent PSHA [16], the latter includes uncertainties in the assumptions used in the source model and extensive subjective engineering judgments used to develop the logic tree. (The trapezoid-shaped source model, which does not satisfactorily agree with spatial seismicity, was developed by overlapping a number of different types of data including neotectonic architecture, Bouguer gravity anomalies, topography map, etc, in a Geography Information System, but this unorthodox, in-house approach was not explained and justified). These undisclosed details make it impossible to verify or to independently perform such computations. In contrast, this paper presents an analysis which is repeatable and requires only a few justifiable assumptions (i.e., magnitude and distance thresholds).

Klugel also commented that a robust seismic hazard estimate should be verified by empirical control. Using the maximum SOPGA from the past 110 years as an empirical indicator, the annual exceedance probability would be approximately 0.9% ($=1/110$). At NPP sites 1 through 4 the maximum SOPGAs are 0.332g, 0.404g, 0.292g and 0.284g, respectively. Fig. 7 shows that the

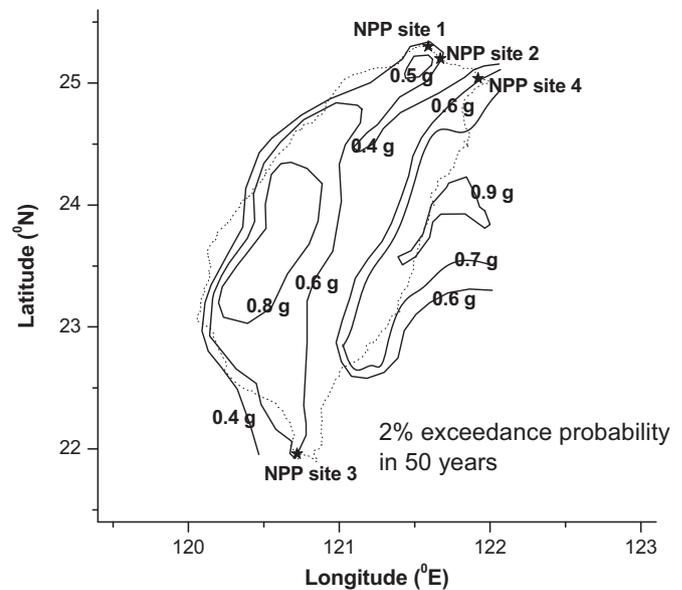


Fig. 8. Seismic hazard map in 2% exceedance probability within 50 years estimated by a recent PSHA study for Taiwan (after Cheng et al. [16]).

best-estimate exceedance probabilities are 0.3%, 0.3%, 1.3% and 2.1%, for those SOPGA values, close to the empirical exceedance probability of 0.9%. The estimations from the study agree reasonably well with the empirical control, providing confidence and support for the approach and result. In contrast, Fig. 8 shows the hazard map in the return period of 2475 years from the PSHA discussed earlier [16], with 0.04% ($=1/2475$) annual exceedance probability. At NPP site 2, the PGA estimate is very close to the empirical maximum (i.e., 0.4g) considering the past 110 years, but its exceedance probability (0.04%) is not in good agreement with the empirical control (0.9%).

The aforementioned comparisons are not intended to degrade the PSHA method and the recent PSHA study for Taiwan, which applied non transparent in-house procedures for the input characterizations. But this discussion points to the fact that no matter what method is used, accountability in results should not be judged by the complexity during analysis, but instead by factors such as transparency, verifiability, and traceability [17,18]. With these prerequisites in place, all types of seismic hazard analysis (e.g., PSHA, DSHA, statistics-oriented probabilistic analysis) are worth considering for developing site-specific earthquake resistant design, considering that there are challenges associated with all types of seismic hazard analysis [17].

8.3. Data availability

"Thanks" to the active seismicity around the study region, the 438 $M_w \geq 5.5$ earthquakes since 1900 seems to be representative of the regional seismicity, not to mention that the largest event within this catalog registered at M_w 9.26, close to the largest earthquake recorded in history and possibly approaching the largest theoretical event that could be justified [21,22]. Statistically speaking, with such a large sample size the statistical attributes should not change significantly owing to a few additional observations. In other words, the seismic hazard and their exceedance rate/probability estimated by this statistics-oriented probabilistic approach are less sensitive to future extreme events, meeting another criterion referred to as the robustness recommendation [18].

It should be noted, that this analysis does require the availability of an adequate quantity of samples. This seismic hazard approach is not suggested for use in inactively seismic regions.

8.4. Site-specific earthquake resistant design

Although different in methodology and result, our analysis and the referred PSHA show one thing in common; seismic hazard is site-dependent or site-specific, even for a relatively small region like Taiwan. As the recent technical reference from the U.S. Nuclear Regulatory Commission suggests [11], earthquake resistant design should evolve into being site-specific, especially for critical structures.

8.5. On-site PGA measurement

In a lack of instrumentation at the four study sites covered by the earthquake detection network in Taiwan, exact on-site PGA measurements are not available. We must agree that such a statistical method in this paper will become more straightforward in seismic hazard evaluation using on-site measurements directly, compared to the use of back-calculated data through ground motion models that has been shown. However, any seismic hazard analysis involving the use of ground motion models introduces questions about models' suitability and reliability. Here we recommend that on-site PGA measurements be implemented at sites occupied by critical structures. Measured ground motions would be valuable for cross-checking the predicted motions with the direct recorded motions. This would allow recalibrating for new ground motion models or better evaluating the suitability of the existing models.

9. Conclusion

This paper studied the statistical attributes of SOPGA back-calculated from intense seismicity around four study sites in Taiwan. The study shows that when an adequate number of sample events are available, the SOPGA might be modeled by a double lognormal distribution. This model can then be used to develop a probabilistic framework in site-specific seismic hazard assessments and provides an approach which has transparency, verifiability, and traceability. In addition, for the four sites evaluated in this study, the seismic hazards are noticeably different, indicating the necessity of site-specific earthquake resistant design.

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