New evidence and perspective to the Poisson process and earthquake temporal distribution from 55,000 events around Taiwan since 1900

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Abstract: Earthquake prediction is by all means controversial and challenging, given the fact that some recent catastrophic earthquakes went unpredicted. Not surprisingly, statistical approaches have been utilized to model the earthquake randomness in time or space. One of the suggestions is that the earthquake’s temporal probability distribution should follow the Poisson model, which is suitable for rare events by definition. As a result, the customarily-used hypothesis should be at large associated with our prior judgment that earthquakes are rare, but not a result of abundant quantitative evidence or theoretical derivation. Therefore, this study aims to offer new empirical evidence to the hypothesis based on the 110-year-long earthquake data around Taiwan. From the series of statistical tests, the first statistical inference is indeed in line with the model’s proposition: the level of fitting between observation and theory is better for earthquakes
with a lower mean rate. To be more specific, it shows that the Poissonian hypothesis applied to \( M_L \geq 3.0 \) earthquakes around Taiwan with a mean annual rate as high as 1,600 is clearly rejected, but as far as \( M_L \geq 7.0 \) earthquakes with a mean rate of 0.35 per year are concerned, the same hypothesis is statistically accepted for modeling their temporal randomness. Also according to the tests on a variety of conditions, the annual rate around 0.1 per year (or 10-year return period) was suggested as a reasonable, empirical estimate as for the Poissonian rareness. Accordingly, from a practical point of view, it should be a robust analytical presumption to use the Poisson model in daily earthquake engineering analyses because the return period of design earthquakes is longer than 10 years if not much longer.

**Keywords:** Poisson model, earthquake temporal distribution, earthquakes in Taiwan, statistical analyses

**Introduction**

As a result of natural randomness, by no means can earthquakes be perfectly predicted by any of a mathematical model. But from the statistical point of view, some models are considered more reasonable than others in simulating earthquake randomness in time or space. For example, a uniform distribution is considered logical for modeling the earthquake’s spatial randomness within a predefined seismic zone (McGuire and Arabasz 1990; Kramer 1996), and this presumption is employed in *Probabilistic Seismic Hazard Analysis (PSHA)*, or known as the Cornell-McGuire approach (Cornell 1968;
McGuire and Arabasz 1990) for distinguishing it from other seismic hazard analyses also under a probabilistic framework (Algermissen et al. 1982; Wang et al. 2012a).

On the other hand, earthquake frequency or its temporal distribution is usually considered following the Poisson process, under which the earthquake rate is a Poissonian random variable and the recurrence time follows an exponential distribution. This hypothesis has been customarily employed in earthquake studies of different scopes (Ashtari Jafari, 2010; Wang et al. 2012b; Weichert 1980). But by definition, the Poisson process is appropriate for simulating the occurrence probability of rare events in time (Ross 2002; Suhir 1997). However, to our best knowledge, the quantitative support of applying this statistical model to earthquakes is little except the statistical study based on the 40-year-long (1930s to 1970s) earthquake data from Southern California (Gardner and Knopoff 1974). Therefore, the Poisson model used in earthquake probabilistic analysis should be at large associated with our prior, logical judgment that earthquakes are rare events, in particular large ones.

One underlying attribute of the Poisson process is the time-independent recurrence probability, or the so-called memory-less effect owing to the mathematical nature of the exponential distribution. However, as the comment of Kagan and Jackson (1999), there was still room for discussing whether or not the earthquake recurrence is time-independent on the basis that some non-Poissonian renewal models, such as the lognormal distribution (Nishenko and Buland, 1987), were found offering a better fit than the memory-less exponential distribution in simulating the earthquake’s recurrence interval. Similar studies (Utsu, 1984; Ellsworth, 1995) were conducted to search for statistical models, either time-independent or time-dependent, with better model
goodness in simulating the earthquake’s temporal randomness. It is worth noting that those models suggested are a result of statistical case studies, supported with the empirical evidence from the statistical point of view.

The elastic-rebound model (Reid, 1910) is the underlying framework of earthquake mechanism. That is, the strain (or strain energy) accumulated in rock will be released and cause earthquakes when it reaches the critical point. Adding the earthquake physics to the development of earthquake recurrence models, the Brownian model was recently proposed (Ellsworth et al., 1999; Matthew et al., 2002). In short, different from those statistical models suggested and supported with empirical, statistical findings, the Brownian model is a “mechanical-statistical” framework. The mechanical component is on the basis of the elastic-rebound model, and, therefore, the earthquake recurrence is a function of strain accumulation and strain relaxation in time. Next, the statistical attributes of the two strain variables or the model’s parameters can be best estimated or calibrated with earthquake data.

The Bayesian approach has been increasingly applied to engineering practices of many types, such as evaluating the reliability of piles with the manufacturer’s claim and in-house tests (Zhang et al., 2006), adjusting the creep model for concrete with experimental results (Raphael, et al., 2009). To sum up, different from the classic statistical analysis purely drawing conclusion from samples, the Bayesian approach is a framework combining the prior information and the likelihood function (i.e., observation) as to obtain and use the resulting posterior estimate in follow-up analyses. Therefore, the Bayesian approach should also have a niche in earthquake recurrence studies by considering the Poisson hypothesis (or others) as a prior updated with earthquake
observation. Ogata (1999) conducted a pioneering study in this regard, proposing the Bayesian, posterior models given a few prior statistical distributions. This certainly is a relatively new option for earthquake recurrence forecasting in future.

As a result, the underlying scope of this study is to revisit the relationship between the Poisson process and earthquake recurrence in time with the 110-year-long earthquake data around Taiwan. Moreover, some discussion is given in an attempt to bridge the Poisson model and daily earthquake engineering practices with the new finding based on a total of 21 statistical tests. Along with the results and interpretations, the background relating to the earthquake catalog and the declustered procedure is also given in this paper, as well as the statistical goodness-of-fit test.

Earthquake catalog for Taiwan and earthquake declustering

Figs. 1 and 2 show the spatial and temporal distributions of the declustered earthquakes around Taiwan since year 1900. A total of more than 55,000 main shocks in local magnitude ($M_L$) are contained in this 110-year-long catalog. The spatiotemporal double-link cluster analysis (Wu and Chiao, 2006) was used in this study, which was a derivative of the single-link cluster analysis proposed by Davis and Frohlich (1991). With the linking parameters of 5 km and 3 days, the “dependent” events in the catalog associated with $M_L \geq 4.5$ main shocks were removed. Note this pair of declustering parameters is customarily used in the processing of the “raw” earthquake data provided by the Central Weather Bureau Seismic Network Taiwan (Wu and Chiao, 2006; Wu and Chen, 2007; Wu et al., 2008), and such a declustered catalog was then used in a few earthquake studies for Taiwan (Wang et al., 2011; Wang et al., 2012a).
Not surprisingly, this catalog is incomplete for small earthquakes in the earlier period of recording as shown in Fig. 2a. Wang et al. (2011) suggested that this catalog should be complete for $M_L \geq 3.0$ earthquakes after 1978. On the other hand, as far as $M_L \geq 5.5$ earthquakes are concerned, this catalog can be considered a complete recording from the very beginning. Also note that the data in year 1951 is “strange” with that many $M_L \geq 5.5$ earthquakes recorded. Since the same declustering procedure was employed, the possibility that the strange data caused by improper declustering can be excluded. Therefore, this “spike” could be a result of other procedural errors, or simply a result of natural randomness.

Since we can not provide support to either of the two possibilities (natural randomness or procedural error) causing the “strange year,” we adopted a practical solution to examine their influence by performing two tests, with and without the data in 1951. More details of the sensitivity study are given in the following.

**Overview of statistical goodness-of-fit analysis**

Statistical goodness-of-fit tests are a method that can be used to examine whether a random variable follows a probability distribution of interest. A few approaches, such as probability papers, the Chi-square test, the Kolmogorov-Smirnov (K-S) test, have been developed. It must be noted that the use of probability papers is not an entirely quantitative approach since it somehow relies on our “judgment” to interpret whether the samples plotted on the paper look like a straight curve or not. If it is, the samples are considered following the underlying distribution tested, and otherwise.
As a result, “quantity-based” tests that are objective are more used in statistical goodness-of-fit studies. Based on the same earthquake catalog, Wang et al. (2011, 2012a), for example, learned that with the K-S tests, the annual maximum earthquake magnitude since 1900 around Taiwan statistically follows the lognormal distribution or Gamma distribution, and the semi-observed PGA follows a double-lognormal distribution. One of the reasons those studies preferred the K-S test to the Chi-square test is to avoid the necessary, subjective selection of a bin size in the making of histograms. Although the influence of bin sizes on the Chi-square test is usually irrelevant to the result in most situations, the K-S test, no involvement of this subjective selection, is after all more objective than the Chi-square test in this regard. However, because the underlying technical challenge in converting the chronicle earthquake data listed in the catalog into a regular “K-S” format for testing the Poisson hypothesis remains unsolved (by us), the Chi-square test was employed in this study. More explanations to the extra, necessary step in data reduction are given in the following.

Proposed in the 1900’s (Pearson, 1900), the essence of the Chi-square test is to compare the difference between theoretical and observational frequencies presented in histograms, as shown in the schematic diagram in Fig. 3. The difference is then characterized by the so-called Chi-square value, expressed as follows:

$$\chi^2_{\text{sample}} = \sum_{i=1}^{n} \frac{(e_i - o_i)^2}{e_i}$$

(1)
where \( e_i \) and \( o_i \), in this study, denote theoretical (Poissonian) and observational earthquake frequencies in years, and \( n \) is the total number of bins in the histogram. In the following analyses, the summation of the frequency in the histogram must be 110 years (or 109 years in the sensitivity study). Note that the degree of freedom of this random variable (i.e., \( \chi^2 \)) is equal to \( n - 1 - p \), where \( p \) is the number of parameters governing the probability distribution. Therefore, the degree of freedom is equal to \( n - 2 \) in this study because the Poisson model is governed by one parameter, i.e., the mean rate.

Given the degree of freedom, the probability function for this random variable can be obtained, so as the critical value corresponding to a specific right-tail probability as shown in Fig. 4. The logic of the Chi-square test is that \( \chi^2_{\text{sample}} \) could be anywhere on the curve with respective probabilities. But as shown in Fig. 4, there is a 1-to-99 chance that \( \chi^2_{\text{sample}} \) is greater and less than the critical value. Therefore, the decision-making rule in the Chi-square test is that despite that small right-tail probability and \( \chi^2_{\text{sample}} \) still in the rejection region (i.e., \( \chi^2_{\text{sample}} > \chi^2_{\text{critical}} \)), the statistical inference is that the prior assumption is most likely not sustainable. In other words, the hypothesis is statistically accepted when \( \chi^2_{\text{sample}} \) is located in the broad, acceptance region. As a result, the small right-tail probability is referred to as level of significance (\( \alpha \)) in goodness-of-fit tests, and it is usual to adopt 1% (or 5%) in the tests.

**The Poisson distribution and corresponding Chi-square test**

The probability mass function for a Poissonian (discrete) random variable (\( X \)) is expressed as follows:
Pr(X = x | \lambda) = \frac{\lambda^x e^{-\lambda}}{x!} \quad (2)

where \lambda is the mean rate or the mean value of X. Given the duration of 110 years (the same as the duration of this catalog), Fig. 3 also shows an example of the theoretical earthquake frequency in years (= probability × 110) with \lambda = 1. Note that the calculation of the mean rate is straightforward by counting the number of earthquakes divided by the years of recording.

However, it is not that straightforward for the making of the observational frequency in the Poissonian form with the earthquake data listed in a chronicle catalog. An extra step in data conversion or data reduction is needed and it was summarized as follows: 1) separate the catalog into a number of yearly-based sub-catalogs, 2) count the annual rate of the earthquake of interest in each sub-catalog, and 3) combine the interim data into resulting Poissonian statistics. (This necessary procedure that we referred to is given in a statistics textbook (Ang and Tang, 2007). It is worth noting that although Gardner and Knopoff (1974) conducted a similar work to this study, the “secrets” were not explicitly given with only the two resulting Chi-square values reported.)

Therefore, the procedure of this hypothesis test can also be summarized as follows: 1) calculate the mean annual rate of earthquakes from the catalog, 2) calculate the theoretical (Poisson) frequency in years with the mean annual rate, 3) develop the observational frequency in the same form with the extra step in data reduction applied to such chronicle data, 4) calculate the Chi-square value based on the theoretical and
observational histograms, and 5) accept or reject the hypothesis from a statistical point of view.

Results

- **Large-area condition and sensitivity study**

  Given the mean rate of $M_L \geq 3.0$ earthquakes around 1,600 per year since 1978, Fig. 5a shows the theoretical earthquake frequency in years calculated with the Poisson distribution. In contrast, Fig. 5b shows the observational frequency by counting the earthquake in the catalog with the specific procedure mentioned. Not surprisingly, under such a high rate, the two curves are fundamentally different, which leads to a statistical inference that the temporal distribution of $M_L \geq 3.0$ earthquakes around Taiwan should not follow the Poisson process, a reflection to the model’s proposition suitable for rare events.

  After realizing that the frequent, $M_L \geq 3.0$ earthquakes around Taiwan are not a Poissonian random variable, we carried out more tests with different magnitude thresholds from 5.5 to 7.0. By observing the changes in Fig. 6, one can perceive that the level of fitting between theory and observation is better with larger magnitude thresholds, or with lower annual mean rates. Once again, this empirical evidence reflects the model’s proposition that it should not be used unconditionally. To be more specific, the empirical finding shows that $M_L \geq 7.0$ earthquakes with a mean rate equal to 0.35 per year since 1900 are considered rare enough to be modeled by the Poisson distribution, but not for the four magnitude thresholds (i.e., 3.0, 5.5, 6.0, and 6.5) based on the same earthquake catalog.
As mentioned, the data in year 1951 seem strange and the cause remains unclear. Therefore, a sensitivity study was performed and the results were shown in Fig. 7. Basically, the curves derived with the 109-year-long data are not significantly different from those using the 110-year-long data shown in Fig. 6. More importantly, the same statistical inference is attained: Among different magnitudes, only for $M_L \geq 7.0$ earthquakes the Poissonian hypothesis is not rejected by the statistical test, and the improved level of fit can be sensibly perceived with larger magnitudes or lower rates.

According to the sensitivity test, in the following analyses on small-area condition we used the 110-year-long data as it is. It is not only because the influence of the strange data in 1951 was found irrelevant to the statistical inference, but it is also on the consideration that the data could be simply a result of natural randomness.

- **Small-area condition**

  Because the aforementioned analyses are on a large-area condition (i.e., 300 km by 400 km), to further attend to the objective of this study, we carried out 16 additional analyses on small-area conditions. Fig. 8 shows the location and size of the eight zones based on the up-to-date seismic source model for Taiwan (Cheng et al., 2007), which has been used in a few seismic hazard analyses (Cheng et al., 2007; Wang et al., 2012c). The reason using this source model is to achieve some objectivity in our test, instead of subjectively selecting a zone in a size and location that could contribute to a “preferable” outcome and suggestion.

  With the same tests but on the small-area condition, the statistical inference is “the same” and “different” from those on the large-area condition. First, the difference is
that $M_L \geq 5.5$ and $M_L \geq 6.0$ earthquakes on the small-area condition become more likely a Poissonian random variable than they are within a broader area. To be more specific, five out of eight tests show that $M_L \geq 5.5$ earthquakes in respective zones are a Poissonian variable, but such a hypothesis was statistically rejected given a large-area condition. Similarly, as far as $M_L \geq 6.0$ earthquakes are concerned, five out of six tests (there are no $M_L \geq 6.0$ earthquakes in two zones) show that such earthquakes in respective zones should follow the Poisson distribution, which again is a contradiction to the earlier statistical finding given in the test considering a large area. But on the other hand, the message behind the two series of tests is also the same. That is, the Poisson hypothesis for the earthquake’s temporal distribution is more acceptable with a lower mean rate per unit time.

It is worth noting that the findings in this case study are also on the same page with that study using earthquake data in Southern California (Gardner and Knopoff 1974), which suggested that the $M_L \geq 3.8$ earthquakes in California with a mean rate of 0.34 per 10 days statistically follow the Poisson distribution. Although that study did not test the hypothesis on an annual basis with an equivalent mean rate equal to 13 per year ($= 0.34 \times 36.5$), according to our empirical finding, we will be very surprised if the same hypothesis is not rejected for the same earthquake magnitude on an annual basis. As a result, the two studies indeed share the same information to the relationship between the Poisson distribution and earthquakes: The mean rate per unit time is the key parameter to the hypothesis; even though small earthquakes are frequent on an annual basis within a broad region, they could be a Poissonian variable considering a small time window or a small zone.
Discussion: the decision making for earthquake engineering

With the new evidence to the relationship between the earthquake’s temporal distribution and the Poisson model, one thing is clear: Earthquakes under a specific space or time condition must be rare enough to fulfill this model’s prerequisite. But the question followed is how to judge whether or not the mean rate of interest is statistically rare enough as to properly use of this statistical model. Based on this earthquake study, this question is hard to be answered perfectly given the statistical significance being governed by a few variables (e.g., zone size, time window, natural randomness, etc.) in addition to the underlying mean rate. As a result, we proposed a logical thinking to answer this question from the engineering point of view. After all, the Poisson hypothesis for earthquakes is most likely used in earthquake engineering designs or large-size earthquake forecasts.

According to the series of statistical tests, we found that as mean rate less than 0.35 per year, the Poisson model was not statistically rejected, except for Zone P shown in Fig. 9h. Moreover, the tests also show that the events with annual mean rate around or less than 0.1 per year are statistically Poissonian, without exception. Accordingly, with the empirical finding based on the earthquake data around Taiwan, the mean rate around 0.1 per year (or per unit time) should be a logical estimate as for the Poissonian rareness, which corresponds to a return period in 10 years.

The message linking this empirical finding to daily earthquake engineering becomes clear: It is a robust analytical presumption to use the Poisson model in earthquake engineering, since the design earthquake is most likely associated with a
return period much longer than 10 years. For example, the design earthquake for safety-related structures at nuclear power plants is prescribed in a 10,000-year return period if not 100,000 years (USNRC, 2007). Moreover, for less critical structures such as residential buildings, the design earthquake is, for example, in a 500-year return period based on the technical reference local to the study region (Construction and Planning Agency Taiwan, 2005).

Therefore, it is more of a scientific question if there is a better framework than the Poisson model to explain and simulate the earthquake’s temporal distribution, in particular those with high rates. Such studies could be crucial to the further understanding of the earthquake’s behavior or mechanics. But on the other hand, based on this study it is a robust engineering decision and presumption to confidently use this statistical model in earthquake engineering practices, because the return period of design earthquakes is longer than 10 years, if not much longer.

Conclusions

This paper presents new empirical evidence to the relationship between the earthquake’s temporal randomness and the Poisson model, based on a series of statistical analyses examining the earthquake data since 1900 around Taiwan. The result shows that the Poisson model should not be used unconditionally, but the model’s goodness is strongly related to the mean rate of earthquakes of interest, as the model’s proposition suitable for rare events by definition. For example, this study shows that the Poisson hypothesis for $M_L \geq 3.0$ earthquakes in Taiwan with a mean rate as high as 1,600 per year is by no means a Poissonian given the fundamental difference between theory and
observation. But the same hypothesis for $M_L \geq 7.0$ earthquakes with a mean rate of 0.35 per year is not rejected otherwise.

Also according to the tests, the critical mean rate was empirically suggested around 0.1 per year, as for the Poisson model being statistically accepted in simulating the earthquake’s temporal distribution. In other words, a logical engineering decision could be as follows: This statistical model is robust for daily earthquake engineering practices, because the return period of design earthquakes is longer than 10 years, if not much longer.

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References


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**Fig. 1.** The earthquake’s spatial distribution around Taiwan since year 1900

**Fig. 2.** The earthquake’s temporal distributions around Taiwan since year 1900

**Fig. 3.** A schematic example of the Chi-square test on the Poisson variable by comparing the difference between theory and observation in the histogram

**Fig. 4.** The schematic diagram showing some more essence of the Chi-square test; given that small right-trial probability but $\chi^2_{\text{sample}}$ still in the rejection region, the hypothesis is not accepted

**Fig. 5.** Observational and theoretical (Poisson) frequencies in years for $M_L \geq 3.0$ earthquakes around Taiwan since year 1978

**Fig. 6.** Theoretical and observational earthquake frequencies on four magnitude conditions; the level of fitting increases with a lower mean rate for larger earthquakes

**Fig. 7.** The result of the sensitivity study with 109-year-long earthquake data without the data in year 1951

**Fig. 8.** Eight seismic source zones used in this study, which is part of the up-to-date seismic source model for Taiwan (after Cheng et al. 2007)

**Fig. 9.** Observational and theoretical earthquake frequencies on a small-area condition
Fig. 1. The earthquake’s spatial distribution around Taiwan since year 1900
Fig. 2. The earthquake’s temporal distributions around Taiwan since year 1900

(a) $M_L \geq 3.0$
- mean rate = 1,611 after 1978

(b) $M_L \geq 5.5$
- mean rate = 6.2
**Fig. 3.** A schematic example of the Chi-square test on the Poisson variable by comparing the difference between theory and observation in the histogram.

$$\chi^2_{sample} = \frac{\sum_{i=1}^{n} (e_i - o_i)^2}{e_i}$$

$$DOF = n - 2$$
Decision rule of the Chi-square test:
when $\chi^2_{\text{sample}}$ in the acceptance region, the hypothesis is not rejected, and otherwise

$$\Pr(\chi^2 > \text{critical}) = \text{level of significance}$$
(e.g., 1%)

Fig. 4. The schematic diagram showing some more essence of the Chi-square test; given that small right-trial probability but $\chi^2_{\text{sample}}$ still in the rejection region, the hypothesis is not accepted.
Fig. 5. Observational and theoretical (Poisson) frequencies in years for $M_L \geq 3.0$ earthquakes around Taiwan since year 1978.
\begin{figure}[h]
\centering
\begin{subfigure}{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{fig_a.png}
\caption{$M_c \geq 5.5$}
\end{subfigure}
\begin{subfigure}{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{fig_b.png}
\caption{$M_c \geq 6.0$}
\end{subfigure}
\begin{subfigure}{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{fig_c.png}
\caption{$M_c \geq 6.5$}
\end{subfigure}
\begin{subfigure}{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{fig_d.png}
\caption{$M_c \geq 7.0$}
\end{subfigure}
\end{figure}

\textbf{Fig. 6.} Theoretical and observational earthquake frequencies on four magnitude conditions; the level of fitting increases with a lower mean rate for larger earthquakes.
Fig. 7. The result of the sensitivity study with 109-year-long earthquake data without the data in year 1951
Fig. 8. Eight seismic source zones used in this study, which is part of the up-to-date seismic source model for Taiwan (after Cheng et al. 2007)
(a) Zone N, ML >= 5.5
Mean annual rate = 0.52
χ² value = 40.8
Critical value = 11.34,
given α = 1% and DOF = 3

(b) Zone N, ML >= 6.0
Mean annual rate = 0.20
χ² value = 6.66
Critical value = 9.21,
given α = 1% and DOF = 2

(c) Zone E, ML >= 5.5
Mean annual rate = 0.11
χ² value = 4.15
Critical value = 9.21,
given α = 1% and DOF = 2

(d) Zone E, ML >= 6.0
Mean annual rate = 0.06
χ² value = 3.38
Critical value = 9.21,
given α = 1% and DOF = 2

(e) Zone O, ML >= 5.5
Mean annual rate = 1.26
χ² value = 118
Critical value = 13.28,
given α = 1% and DOF = 4

(f) Zone O, ML >= 6.0
Mean annual rate = 0.39
χ² value = 2.43
Critical value = 11.34,
given α = 1% and DOF = 3
(g) Zone P, ML >= 5.5
Mean annual rate = 0.67
$\chi^2$ value = 23.2
Critical value = 11.34,
given $\alpha = 1\%$ and DOF = 3

(h) Zone P, ML >= 6.0
Mean annual rate = 0.22
$\chi^2$ value = 24.3
Critical value = 9.21,
given $\alpha = 1\%$ and DOF = 2

(i) Zone F, ML >= 5.5
Mean annual rate = 0.07
$\chi^2$ value = 0.32
Critical value = 6.63,
given $\alpha = 1\%$ and DOF = 1

(j) Zone F, ML >= 6.0
Mean annual rate = 0.03
$\chi^2$ value = 0.04
Critical value = 6.63,
given $\alpha = 1\%$ and DOF = 1

(k) Zone I, ML >= 5.5
Mean annual rate = 0.09
$\chi^2$ value = 7.16
Critical value = 9.21,
given $\alpha = 1\%$ and DOF = 2

(l) Zone I, ML >= 6.0
Mean annual rate = 0.04
$\chi^2$ value = 0.08
Critical value = 6.63,
given $\alpha = 1\%$ and DOF = 1
Fig. 9. Observational and theoretical earthquake frequencies on a small-area condition.