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A first-order second-moment calculation for seismic hazard assessment with the consideration of uncertain magnitude conversion

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Abstract. Earthquake size can be described with different magnitudes for different purposes. For example, local magnitude M_L is usually adopted to compile an earthquake catalog, and moment magnitude M_w is often prescribed by a ground motion model. Understandably, when inconsistent units are encountered in an earthquake analysis, magnitude conversion needs to be performed beforehand. However, the conversion is not expected at full certainty owing to the model error of empirical relationships. This paper introduces a novel first-order second-moment (FOSM) calculation to estimate the annual rate of earthquake motion (or seismic hazard) on a probabilistic basis, including the consideration of the uncertain magnitude conversion and three other sources of earthquake uncertainties. In addition to the methodology, this novel FOSM application to engineering seismology is demonstrated in this paper with a case study. With a local ground motion model, magnitude conversion relationship and earthquake catalog, the analysis shows that the best-estimate annual rate of peak ground acceleration (PGA) greater than 0.18 g (induced by earthquakes) is 0.002 per year at a site in Taipei, given the uncertainties of magnitude conversion, earthquake size, earthquake location, and motion attenuation.

1 Introduction

The size of earthquakes can be portrayed with different magnitudes, such as local magnitude M_L and moment magnitude M_w. For example, the 1999 Chi-Chi earthquake in Taiwan reportedly had a local magnitude of 7.3 and a moment magnitude of 7.6. Understandably, the difference is attributed to definitions and measurements. For local magnitude M_L, it is based on the largest amplitude on a Wood–Anderson torsion seismograph installed at a station 100 km from the earthquake epicenter (Richter, 1935). In contrast, moment magnitude M_w is related to the seismic moment of earthquakes, the product of fault slips, rupture areas, and the shear modulus of the rock (Keller, 1996).

Nowadays, local magnitude M_L is commonly adopted by earthquake monitoring agencies, mainly because the measurement is relatively straightforward and its indication to the shaking of buildings is robust (Kanamori and Jennings, 1978). For example, an earthquake catalog complied by the Central Weather Bureau Taiwan is on an M_L basis (e.g., Wang et al., 2011). On the other hand, moment magnitude M_w that is not subject to the so-called magnitude saturation is commonly adopted in the developments of ground motion models, for a more precise ground motion prediction (Wu et al., 2001; Campbell and Bozorgnia, 2008; Lin et al., 2011).

Some empirical models have been suggested for magnitude conversion (Das et al., 2012; Wu et al., 2001). For example, based on the earthquake data around Taiwan, Wu et al. (2001) suggested an empirical relationship between M_L and M_w as follows:

$$M_\mathrm{L} = 4.53 \times \ln(M_\mathrm{w}) - 2.09 \pm 0.14, \tag{1}$$

where the term ± 0.14 is the standard deviation of model error ε. (Based on the fundamentals of regression analysis, the mean value of ε is zero, and it is a random variable following the normal distribution.) In a recent earthquake study for Taiwan (Wang et al., 2013a), this empirical relationship was adopted for magnitude conversion when the units in the earthquake catalog and ground motion model were different. However, it is worth noting that those conversions were performed on a "deterministic" basis disregarding the influence of model error ε. For example, given $M_L = 6.5$, the deterministic estimate for moment magnitude is presented by a single value in $M_w = 6.66$, in contrast to a probabilistic estimate (shown in Fig. 1) displaying the probability distribution of M_w, considering the uncertainty of conversion or the model error of the empirical relationship.

This study aims to consider this additional source of uncertainty to estimate the annual rate of earthquake motion (or seismic hazard) on a probabilistic basis. Instead of following a representative method, this study adopts the first-order second-moment (FOSM) calculation for solving the problem. In addition to the FOSM algorithms detailed in this paper, a case study was performed to demonstrate this novel FOSM application to engineering seismology. This paper also includes an overview of probabilistic analysis, probabilistic seismic hazard analysis (PSHA), etc., and the reason for adopting the FOSM calculation over an existing method for the targeted problem.

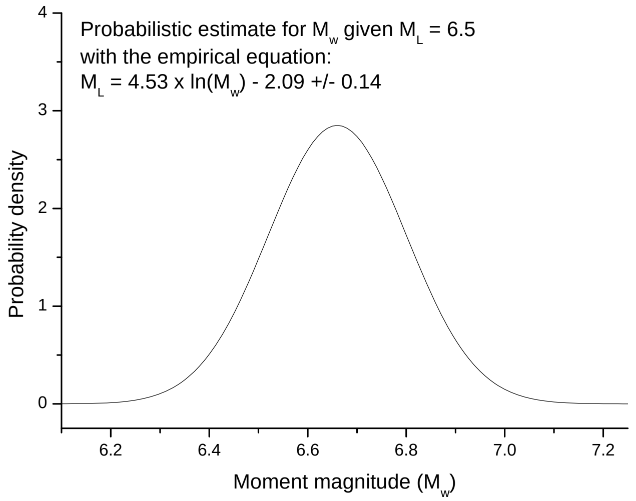

Fig. 1. An example showing the conversion from M_L to M_w on a probabilistic basis.

2 Overviews of probabilistic analysis, deterministic analysis, PSHA, and DSHA

2.1 Probabilistic analysis and deterministic analysis

Probabilistic analysis or deterministic analysis is a general concept for solving a problem on a probabilistic or deterministic basis. In other words, the two are applicable to many subjects, from social science to earthquake engineering. The key difference between probabilistic analysis and deterministic analysis can be demonstrated with the following example: given $Y = A + B$ (where Y, A, and B are all random variables), a deterministic analysis aims to find the mean value of Y with the mean values of A and B but without considering their variability. By contrast, when both mean values and standard deviations (SDs) of A and B are taken into account to solve the mean and SD of Y, the calculation is then referred to as probabilistic analysis.

It is worth noting that the analytical solution for probabilistic analysis is usually non-existent when the function of random variables becomes more complex, even for a simple function like $Y = \log A + B$, where A and B are both random variables uniformly distributed from 0 to 10. For solving such a problem, alternatives such as Monte Carlo simulation (MCS), first-order second-moment (FOSM), and point estimate method are more applicable. For example, by randomly generating 5000 A and B values and substituting them into the governing equation, one can obtain a series of Y values. Accordingly, such an MCS (sample size = 5000) shows that the SD of Y is around 3 for this problem based on 5000 MCS samples.

On the other hand, the problem can be solved with different computations than MCS. For example, the FOSM calculation estimates that the SD of Y for the same problem is equal to 2.94, close to the estimate from MCS. As a result, it is the nature of probabilistic analysis to have different solutions (close to each other) depending on how the computation is performed, especially when the analytical solution is not available.

2.2 Two representative seismic hazard analyses: PSHA and DSHA

Before introducing seismic hazard analysis, it is worth clarifying the definition of earthquake hazard or seismic hazard. Instead of referring to casualty or economic loss, as the word "hazard" might implicate, seismic hazard is related to an earthquake ground motion or its annual rate. For example, deterministic seismic hazard analysis (DSHA) would estimate the seismic hazard of peak ground acceleration (PGA) equal to 0.3 g at the site, and probabilistic seismic hazard analysis (PSHA) would suggest the rate of PGA $> 0.3\,g$ around 0.01 per year.

With a number of case studies reported (e.g., Cheng et al., 2007; Stirling et al., 2011; Wang et al., 2013a), DSHA and PSHA should be the two representative approaches to seismic hazard assessment nowadays. In terms of algorithms, DSHA estimates the seismic hazard given a worst-case earthquake size and location, and PSHA evaluates the annual rate of ground motion with the consideration of the uncertainties of earthquake size, location, and attenuation (Kramer, 1996). In the industry, DSHA has been prescribed by California since the 1970s as the underlying approach to the development of earthquake-resistant designs (Mualchin, 2011).

On the other hand, a recently implemented technical guideline prescribes the use of PSHA for designing critical structures under earthquake conditions (US Nuclear Regulatory Commission, 2007).

Nevertheless, it is worth noting that the two analyses are associated with a specific algorithm, rather than a general framework that probabilistic analysis and deterministic analysis present. As a result, it is understood that "probabilistic analysis applied to seismic hazard assessment" is a framework, in contrast to a specific algorithm (see the Appendix) like probabilistic seismic hazard analysis or the Cornell method. A recent study using a new algorithm to estimate the annual rate of earthquake motion should further explain the difference between the two (Wang et al., 2012a). With the same purpose to estimate seismic hazard by considering earthquake uncertainties in different ways, both the new approach and the conventional PSHA are part of probabilistic analysis to solve the specific problem of engineering seismology. This instance is an analog to the demonstration that was just shown: a probabilistic analysis aiming to estimate the SD of Y governed by $Y = \log A + B$ can be solved with MCS, FOSM, or a few other probabilistic analyses.

Understanding that PSHA presents a specific algorithm, one comes to realize the possibility of applying other computations such as FOSM to seismic hazard assessment as this paper will detail in the following. To avoid confusion, this paper refers to PSHA as the Cornell method (Cornell, 1968) to differentiate it from other probabilistic approaches to earthquake hazard assessment. The reason for not extending the Cornell method for the targeted problem of this study is discussed later in this paper.

3 Methodology

3.1 Overview of FOSM

As previously mentioned, FOSM is one of the common methods to solve the problem of many different subjects. For example, Na et al. (2008) adopted the FOSM calculation to evaluate the influence of variability in the soil's friction angle and shear modulus on the structure's performance. Kaynia et al. (2008) utilized a FOSM framework to estimate the site's vulnerability to landslide with a few sources of uncertainty taken into account, and Jha and Suzuki (2009) used FOSM to evaluate the potential of soil liquefaction on a probabilistic basis. The instance may go on and on, and the message behind it is that the FOSM approach is commonly accepted nowadays for performing a probabilistic computation.

3.2 FOSM algorithms and computations

From the title of the method, one could expect that the Taylor expansion plays an important role in FOSM (Hahn and Shapiro, 1967). Take the simple case of $Y = g(X)$ for example (X and Y are random variables). The Taylor expansion up to the first-order term against constant μ_X (i.e., the mean value of X) can be expressed as follows (Ang and Tang, 2007):

$$Y = g(X) \approx g(\mu_X) + (X - \mu_X)\, g'(\mu_X), \tag{2}$$

where $g' = \frac{\mathrm{d}g}{\mathrm{d}X}$. To derive the mean value of Y, Eq. (2) can be written as the following equation, where E denotes the mean value:

$$E[Y] \approx E\left[g(\mu_X) + (X - \mu_X)\, g'(\mu_X)\right]. \tag{3}$$

Since μ_X is a constant, Eq. (3) can be rewritten as

$$E[Y] \approx g(\mu_X) + g'(\mu_X) E[X] - \mu_X g'(\mu_X). \tag{4}$$

Given $E[X] = \mu_X$, two terms on the right-hand side of Eq. (4) are canceled out, so that the mean value of Y is approximated to $g(\mu_X)$.

Similar derivation can be applied to derive the variance of Y. (Variance is the square of standard deviation.) From Eq. (2), the variance of Y (denoted as $V[Y]$) can be approximated as follows:

$$V[Y] \approx V\left[g(\mu_X) + (X - \mu_X)\, g'(\mu_X)\right]. \tag{5}$$

Since the variance of constants is equal to zero, the variance of Y is as follows:

$$V[Y] \approx V\left[X\, g'(\mu_X)\right] = V[X]\; g'(\mu_X)^2 = \sigma_X^2\; g'(\mu_X)^2, \tag{6}$$

where σ_X is the standard deviation of X.

The same derivation based on the Taylor expansion can be followed and applied to a more complicated case of $Y = g(X_i s)$. For such a function of multiple random variables, the mean and variance of Y related to those of $X_i s$ can be expressed as follows (Ang and Tang, 2007):

$$E[Y] \approx g\left(E[X_1], E[X_2], \cdots, E[X_n]\right) \tag{7}$$

and

$$V[Y] \approx \sum_{i=1}^{n}\left[\left(\frac{\partial g}{\partial X_i}\right)^2 V[X_i]\right] + 2\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\frac{\partial g}{\partial X_i}\frac{\partial g}{\partial X_j}\mathrm{Cov}(X_i,\, X_j)\right) \quad ; \quad \text{for } i < j, \tag{8}$$

where n denotes the number of $X_i s$, and Cov is the covariance between two variables. For the case that any of two input variables are independent of each other (covariance is zero when two variables are independent), the variance of Y can be approximated as follows:

$$V[Y] \approx \sum_{i=1}^{n}\left[\left(\frac{\partial g}{\partial X_i}\right)^2 V[X_i]\right]. \tag{9}$$

As a result, the expressions shown in Eqs. (7)–(9) are the underlying FOSM algorithms for performing a probabilistic analysis.

Since the following case study is a computer-aided analysis, for advancing the calculation of Eq. (9) in a computer, the finite difference approximation of the derivative was followed in this study (US Army Corps of Engineers, 1997). Taking X_1 for example, $\frac{\partial g}{\partial X_1}$ can be approximated as follows:

$$\frac{\partial g}{\partial X_1} = \frac{g(\mu_1+\sigma_1,\ \mu_2,\cdots,\mu_n) - g(\mu_1-\sigma_1,\ \mu_2,\cdots,\mu_n)}{2\sigma_1}, \quad (10)$$

where μ_1 and σ_1 denote the mean and SD of X_1, respectively.

3.3 The governing equations of this study

The governing equation of this study is related to a ground motion prediction equation. Because the following case study is about a site in Taiwan, we directly used a local ground motion model used in recent earthquake studies for Taiwan (Cheng et al., 2007; Wang et al., 2013a) to derive the governing equation:

$$\begin{aligned}\ln \mathrm{PGA} = &-3.25 + 1.075\, M_\mathrm{w} \\ &- 1.723 \ln(D + 0.156 \exp(0.624 M_\mathrm{w})) + \varepsilon_\mathrm{M}, \quad (11)\end{aligned}$$

where D denotes the source-to-site distance in km; ε_M is the model error, whose standard deviation was reported at 0.577 (mean = 0). From Eq. (11), it should be understood that PGA is a variable governed by three variables (M_w, D and ε_M) or their uncertainty.

It must be noted that this model requires moment magnitude M_w to describe the size of earthquakes. Since the input seismicity (described later) is in M_L, magnitude conversion is needed to match the unit M_w prescribed by the ground motion model. To complete the conversion, the relationship between M_L and M_w shown in Eq. (1) was combined with Eq. (11), so that the governing equation of this study becomes

$$\begin{aligned}\ln \mathrm{PGA} &= g\,(M_\mathrm{L}, D,\ \varepsilon, \varepsilon_\mathrm{M}) \\ &= -3.25 + 1.075 \exp\left(\frac{M_\mathrm{L}+2.09+\varepsilon}{4.53}\right) \\ &- 1.723 \ln\Bigg(D + 0.156 \exp \\ &\left(0.624 \exp\left(\frac{M_\mathrm{L}+2.09+\varepsilon}{4.53}\right)\right)\Bigg) + \varepsilon_\mathrm{M}. \quad (12)\end{aligned}$$

Understandably, lnPGA or PGA is now governed by four random variables, including ε of magnitude conversion. Next, the FOSM computations (Eqs. 7–10) are utilized to solve the governing equation (Eq. 12) to obtain the mean and SD of PGA, given those of four variables (M_L, D, ε, and ε_M) appearing in the governing equation.

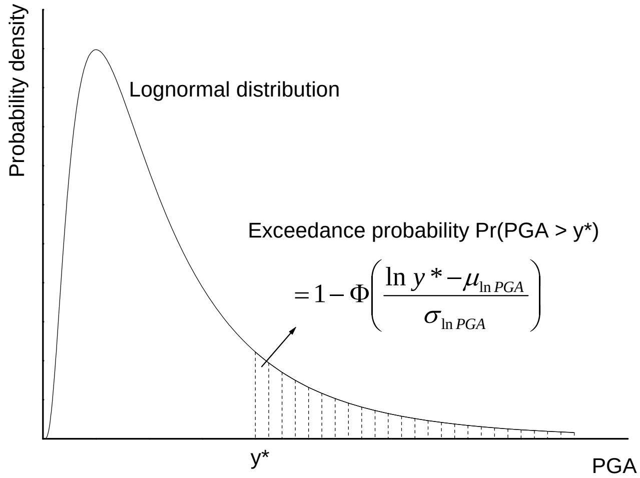

Fig. 2. A schematic diagram showing the PGA exceedance probability given its probability distribution.

3.4 Exceedance probability, earthquake rate, the rate of seismic hazard

In addition to the mean and SD, a suitable probability model is needed to develop the probability density function (PDF) of a random variable. To the best of our knowledge, earthquake ground motion such as PGA is considered a variable following the lognormal distribution (Kramer, 1996), or lnPGA follows a normal distribution. With the three pieces of information, the PDF of PGA can be developed and the probability PGA exceeding a given value $y*$ can be calculated with the fundamentals of probability (Ang and Tang, 2007):

$$\begin{aligned}\Pr(\mathrm{PGA} > y*) &= \Pr(\ln \mathrm{PGA} > \ln y*) \\ &= 1 - \Pr(\ln \mathrm{PGA} \le \ln y*) \\ &= 1 - \Phi\left(\frac{\ln y* - \mu_{\ln \mathrm{PGA}}}{\sigma_{\ln \mathrm{PGA}}}\right), \quad (13)\end{aligned}$$

where Φ denotes the cumulative density function of a standard normal distribution (mean = 0 and SD = 1); $\mu_{\ln \mathrm{PGA}}$ and $\sigma_{\ln \mathrm{PGA}}$ are the mean and standard deviation of lnPGA from solving the governing equation (i.e., Eq. 12). For a better illustration of the calculation of exceedance probability, a schematic diagram is shown in Fig. 2.

Like the Cornell method, the rate of earthquake occurrences (v) is taken into account in this analysis to estimate the annual rate of a given PGA exceedance, denoted as $\lambda_{\mathrm{PGA}>y*}$. Following the representative Cornell method (see the Appendix), the algorithm of this step is simply the product of earthquake rate and exceedance probability:

$$\lambda_{\mathrm{PGA}>y*} = v \times \Pr(\mathrm{PGA} > y^*), \quad (14)$$

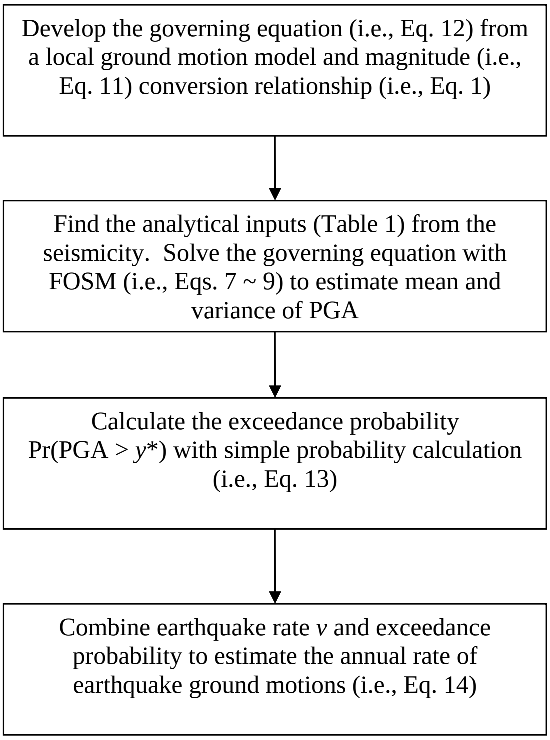

Fig. 3. A chart summarizing the FOSM application of this study to earthquake hazard assessment.

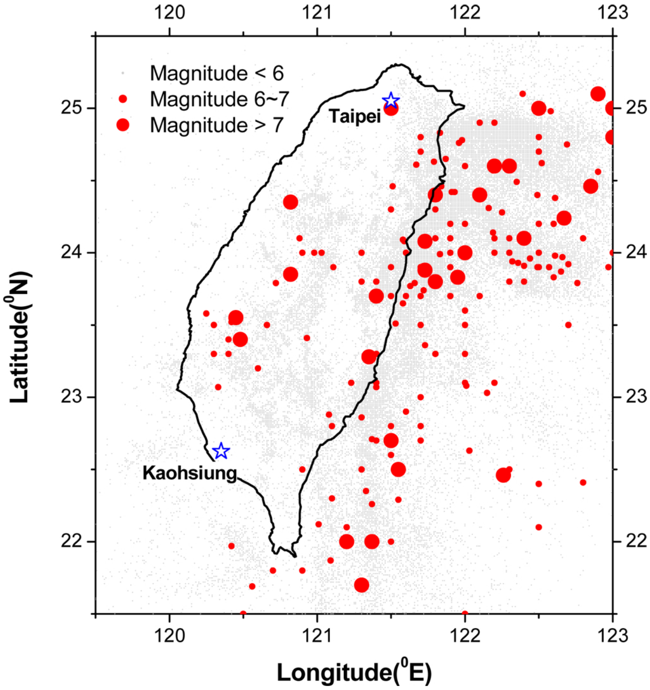

Fig. 4. The seismicity around Taiwan since 1900, based on a catalog containing around 57 000 records.

where the earthquake rate υ can be estimated from a given seismicity, and exceedance probability Pr(PGA $> y^*$) can be obtained with the procedure that was just described.

3.5 Seismicity-based earthquake hazard analysis

Like a recent seismicity-based analysis (Wang et al., 2012a), this study also estimates the annual rate of earthquake motion with the statistics of "major earthquakes" around the site, which are referred to as relatively large earthquakes occurring within a given distance (e.g., 200 km) from the site. In other words, such an analysis is mainly governed by the randomness of the so-called major earthquakes observed in the past 110 yr. It is worth noting that the analytical presumptions about magnitude and distance thresholds are also needed by the Cornell approach, like a PSHA study using a magnitude threshold of $M_L = 5.5$ and a distance threshold of 150 km (Wang et al., 2013a).

Since the two thresholds are the only two engineering judgments needed, such a seismicity-based method is more transparent and repeatable – the key criteria for the so-called robust seismic hazard analysis (more discussion is given in Sect. 5.2). Take seismic hazard studies for Taiwan as an example: a recent discussion pointed out that one assessment was hardly repeatable, owing to the involvements of excessive engineering judgments (Wang et al., 2012b).

3.6 The summary of this FOSM analysis

Figure 3 shows a flowchart summarizing this novel FOSM application to earthquake hazard analysis. The first step is to extract the analytical inputs from a given earthquake catalog, followed by solving the mean and SD of lnPGA, or PGA, in the governing equation. The next step is to calculate PGA exceedance probabilities with simple probability calculations, followed by adding the earthquake rate to estimate the annual rate of seismic hazard.

In addition to magnitude and distance thresholds, another presumption adopted in this analysis is the independence between earthquake variables, which is also assumed and employed in the representative Cornell method. However, it could be possible that large earthquakes tend to recur in some regions, or earthquake size and location are somewhat correlated. As a result, more studies should be worth conducting to examine the independence of earthquake variables, for justifying existing earthquake analyses using such an "independent" presumption without tangible support.

4 Case study

4.1 Earthquake geology and the seismicity around Taiwan

The region around Taiwan is close to the boundary of the Philippine Plate and Eurasian Plate. Such a tectonic

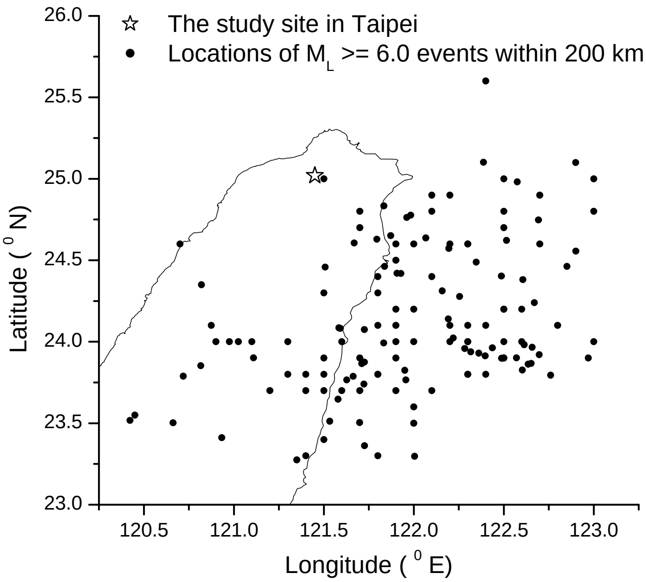

Fig. 5. The large earthquakes above $M_L = 6.0$ within a distance of 200 km from the study site in Taipei.

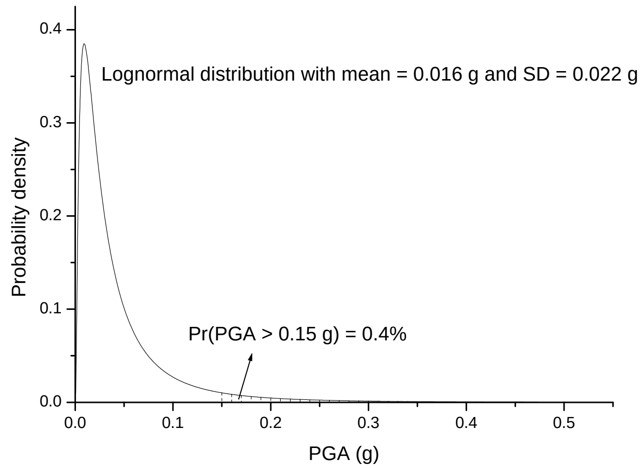

Fig. 6. The expected PGA distribution at the study site, conditional to a random $M_L > 6.0$ event within 200 km from the site.

environment gave birth to the island of Taiwan. Reportedly this orogeny process started around 4 million years ago (Suppe, 1984), and with a plate convergence rate at about $8\,\mathrm{cm\,yr^{-1}}$ as of now (Yu et al., 1997), the tectonic activity around Taiwan should still be active. Such a geological background is the underlying cause of the high seismicity in this region with about 18 000 earthquakes detected every year (Wu et al., 2008). On the other hand, a few catastrophic events (e.g., the 1906 Meishan earthquake, the 1999 Chi-Chi earthquake) have struck the island and have caused severe casualties.

Figure 4 shows the seismicity around Taiwan since 1900 from an earthquake catalog containing around 57 000 events. It is worth noting that this catalog has been analyzed for a variety of earthquake studies, from earthquake hazard assessment (Wang et al., 2013a), to earthquake engineering release (Huang and Wang, 2011), to earthquake statistics study (Wang et al., 2011, 2013b). Understandably, the catalog is not complete for small events. For example, it was pointed out that events above $M_L = 3.0$ in this catalog were complete after year 1978. Only events above $M_L = 5.5$ could be considered complete after 1900.

4.2 The seismic hazard at a site within Taipei

A case study for a site within Taipei, the most important city in Taiwan, was performed with this novel FOSM application. As the summary of this FOSM seismic hazard analysis (Fig. 3), we first extracted major earthquakes around the site from the given seismicity. Figure 5 is an example showing the locations of earthquakes above $M_L = 6.0$ within a distance of 200 km from the site. From the 147 events since 1900, the mean values and standard deviations of magnitude are $M_L = 6.44$ and 0.46, and they are 129 and 39 km for source-to-site distance. Table 1 summarizes the inputs for the analysis, including earthquake rates, magnitude/distance thresholds, and the uncertainties (ε and ε_M) of two empirical equations.

With the input data and the governing equation (Eq. 12), the mean and standard deviation of PGA are equal to 0.016 g and 0.022 g following the FOSM calculations. That is, as an $M_L \geq 6.0$ event occurs PGA is expected to have a mean value = 0.016 g and SD = 0.022 g at this site, given the randomness of earthquake size and location, and the uncertainties or model errors of two empirical relationships. Along with the presumption that PGA follows a lognormal distribution as previously mentioned, Fig. 6 shows its probability density function for this scenario. Accordingly, the probability PGA exceeding 0.15 g is around 0.4 %. With earthquake rate $\upsilon = 1.34$ per year, the rate for PGA > 0.15 g is around 0.005 per year at this site. Such a calculation can be repeated for other ground motion levels to establish a hazard curve as shown in Fig. 7.

4.3 Sensitivity study and the logic-tree analysis

The analysis that was just demonstrated is related to a specific boundary condition (i.e., $m_0 = M_L = 6.0$ and $d_0 = 200$ km), the best engineering judgments that earthquakes with a smaller size or a farther location should not cause damage to structures. (As previously mentioned, such engineering judgments are inevitable for a comparable analysis regardless of the methodology used.) To address such uncertainty, three analyses with different thresholds were performed, followed by a logic-tree analysis (or weight-averaged) to obtain the best-estimate seismic hazard like other studies (e.g., Cheng et al., 2007; Wang et al., 2013a). It

Table 1. Summary of the FOSM analyses for earthquake hazard assessment.

Scenario	Threshold (M_L, km)	Earthquake Frequency		Magnitude (M_L)		Distance (km)		Conversion Error ε_M		Model Error ε		PGA (g)	
		Since 1900	Annual Rate	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
1	(6.0, 200)	147	1.34	6.436	0.463	129.5	39.1	0	0.14	0	0.577	0.016	0.022
2	(6.0, 150)	101	0.92	6.428	0.467	110.6	31.2	0	0.14	0	0.577	0.020	0.026
3	(5.5, 150)	309	2.81	5.917	0.459	109.6	29.5	0	0.14	0	0.577	0.016	0.019
4	(5.5, 100)	108	0.98	5.903	0.444	75.7	18.7	0	0.14	0	0.577	0.009	0.012

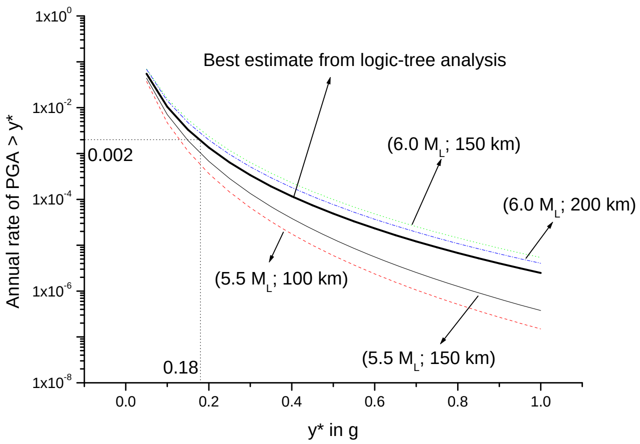

Fig. 7. The sensitivity analysis on the magnitude and distance thresholds and the best-estimate hazard curve for the site with a logic-tree analysis accounting for such uncertainty in the analysis.

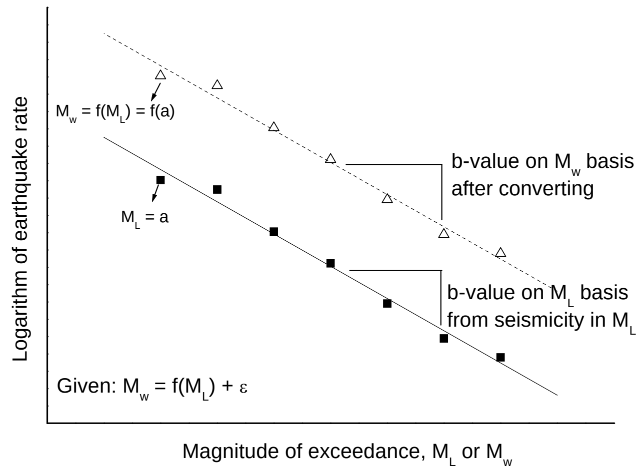

Fig. 8. A schematic diagram illustrating a procedure to obtain a b value on an M_w basis from given seismicity in M_L.

is also worth noting that the four scenarios or boundary conditions of this study were previously adopted in recent seismic hazard studies for Taiwan (Wang et al., 2012a, 2013a).

The sensitivity analysis is summarized in Table 1, with Fig. 7 showing the hazard curves for each of the four scenarios. With an equal weight assigned to each scenario in the logic-tree analysis, the best-estimate hazard curve is also shown in Fig. 7. For example, the occurrence rate for PGA $> 0.18\,g$ was estimated at 0.002 per year. At the same annual rate, we found that this PGA of exceedance (i.e., $0.18\,g$) is comparable to those summarized in a PSHA study for Taiwan (Cheng et al., 2007), reporting a range from $0.15\,g$ to $0.3\,g$ given different source models used.

5 Discussions

5.1 The reason for not extending the existing approach to this study

The so-called b value calibrated with seismicity is needed for the Cornell method to calculate the magnitude distribution (see the Appendix). However, like this study, when different units are encountered in the given earthquake catalog and ground motion models adopted, magnitude conversion must be performed beforehand. Figure 8 is a schematic diagram showing a possible procedure to obtain the b value on an M_w basis, converted from the M_L-based b value. Understandably, this conversion simply applied an empirical model (e.g., Eq. 1) to each data point in the log$N$-M diagram (i.e., Fig. 8), then calculating the slope of converted data points.

However, this conversion is considered a deterministic procedure, because the model error is irreverent to the conversion, or the slope of converted data point is independent of the model error. In other words, when one uses the Cornell method for seismic hazard assessment and encounters the issue of inconsistent units, such an exercise cannot account for the uncertainty of magnitude conversion. This dilemma we encountered then motivated this study aiming to use a new approach to solving the problem of interest: a probabilistic earthquake hazard assessment considers the uncertainty of magnitude conversion, in addition to uncertain earthquake size, location, and motion attenuation.

5.2 Recent discussions on seismic hazard analysis

Although seismic hazard analysis is considered a viable solution to seismic hazard mitigation, some discussion over its methodological robustness has been reported (e.g., Castanos and Lomnitz, 2002; Bommer, 2003; Krinitzsky, 2003; Mualchin, 2011). Mualchin (2005) commented that no seismic hazard analysis should be perfect without challenge, given our limited understanding of the random earthquake process. Moreover, Kluegel (2008) considered that the key to a robust seismic hazard study is a transparent and repeatable process, regardless of methodology.

Like this study, we can expect that more derivatives and algorithms will be developed for estimating earthquake hazards in different ways. In the meantime, unless the seismic hazard estimate (e.g., the rate of PGA $> 0.2\,g = 0.001$ per year) can be verified with field evidence or repeatable testing, we should acknowledge that no seismic hazard assessment is perfect, and that the merit of seismic hazard research should largely reflect its scientific originality, but not by whether or not it follows the state of the practice, given that the virtues of research are to challenge or to create the state of the practice.

6 Conclusion

Earthquake magnitude can be portrayed in different units such as local magnitude and moment magnitude. When the issue of inconsistent units is encountered in an earthquake analysis, the magnitude conversion is needed beforehand. But owing to the model error of empirical relationships, it is understood that the conversion is not at full certainty. This study presents a novel application of first-order second-moment to the problem targeted in this study: a probabilistic earthquake hazard assessment considers the uncertain magnitude conversion in addition to three other sources of earthquake uncertainties. Besides the FOSM algorithm detailed in this paper, a case study was also performed to demonstrate this robust FOSM analysis for earthquake hazard studies. The case study shows, for example, that the rate of PGA $> 0.18\,g$ is 0.002 per year at a site in Taipei, given the statistics of major earthquakes extracted from an M_{L}-based earthquake catalog since 1900, and the model errors of a M_{w}-based ground motion model and magnitude conversion relationship.

Appendix A

The algorithm of the Cornell method

The algorithm of the Cornell method (i.e., PSHA) is as follows (after Kramer, 1996):

$$\lambda(Y > y^*) = \sum_{i=1}^{N_{\mathrm{S}}} \upsilon_i \sum_{j=1}^{N_{\mathrm{M}}} \sum_{k=1}^{N_{\mathrm{D}}} \Pr\left[Y > y^* | m_j, d_k\right] \times \Pr\left[M = m_j\right] \times \Pr[D = d_k], \tag{A1}$$

where N_{S} is the number of seismic sources; N_{M} and N_{D} are the number of data bins in the magnitude and distance probability functions, respectively; υ denotes earthquake rate. Note that the calculation of probability such as $\Pr\left[M = m_j\right]$ in the governing equation indicates that the calculation is performed on a probabilistic basis.

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