A new seismic hazard analysis using FOSM algorithms

J.P. Wang, Yih-Min Wu

Dept of Civil & Environmental Eng., HKUST, Kowloon, Hong Kong
Dept of Geosciences, National Taiwan University, Taipei, Taiwan

ABSTRACT

From recent lessons, it is evident that earthquake prediction is immature and impractical as of now. Under the circumstances, seismic hazard analysis is considered a more practical approach for earthquake hazard mitigation, by estimating the annual rate of earthquake ground motions (or seismic hazard) based on seismicity and other geological evidences. Like other earthquake studies for the high-seismicity region around Taiwan, this study aims to conduct a new seismic hazard assessment for the region using the well-established FOSM (first-order second-moment) algorithm, on the record of 55,000 earthquakes observed in the past 110 years. The new seismic hazard analysis from a different perspective shows that the annual rate for earthquake-induced PGA to exceed the current design value (i.e., 0.23 g) in two major cities in Taiwan should be relatively low, with it no greater than 0.0006 per year. Besides, the FOSM estimates were found very close to those with Monte Carlo Simulation (MCS), mainly because the skewness of the three random variables (i.e., earthquake magnitude, location, and model error) considered in the probabilistic analysis is not very large.

1. Introduction

Since it is challenging to predict an earthquake’s magnitude, location and time, seismic hazard analysis is considered a more practical engineering solution for earthquake hazard mitigation [1]. But before introducing the analysis, it is worth clarifying the definition of seismic hazard in the first place: Rather than casualty or economic loss induced by earthquakes, seismic hazard refers to the annual rate of earthquake ground motions, such as an estimate of PGA > 0.1g = 0.01 per year. In other words, seismic hazard analysis aims to develop a site-specific earthquake-resistant design based on earthquake data (e.g., seismicity) around a site. Nowadays, Deterministic Seismic Hazard Analysis (DSHA) and Probabilistic Seismic Hazard Analysis (PSHA) are the two representative approaches, with many case studies reported in the last few decades [2–5]. On the other hand, new seismic hazard assessments were also reported recently [6–8]. For example, based on earthquake intensity data in the last 500 years, Liu et al. [6] quantified seismic hazards in North China from a different perspective than the conventional PSHA and DSHA.

The region around Taiwan is known for high seismicity. In average, more than two thousand earthquakes with magnitude greater than 3.0 are occurring around this region. Under the circumstances, a variety of earthquake studies for the region were conducted, including seismic hazard assessments [2,3], earthquake early warning [9,10], active fault investigation [11,12], and earthquake risk assessment [13].

As a result, the key scope of this study is to perform a new seismic hazard assessment for this high-seismicity region, using the FOSM algorithm to estimate the annual rate of earthquake motions with the statistics of major earthquakes in the past 110 years. Not only was the new FOSM seismic hazard assessment proved as robust as that with MCS, but the results of the case study are valuable to earthquake-resistant designs in Taiwan.

This paper in the following is organized with an overview of probabilistic analysis, the seismicity around Taiwan, and local ground motion models, followed by the case studies for two major cities in Taiwan. In addition, the seismic hazard estimates from the FOSM computations were compared to those from MCS, showing the two are almost the same, mainly attributed to the low skewness of input random variables (i.e., earthquake magnitude, location, and motion attenuation) considered in this probabilistic analysis.

2. Probabilistic analysis

Unlike deterministic analysis, usually the analytical solution of a probabilistic analysis is difficult to develop. As a result, several algorithms were developed to solve a probabilistic analysis,
including MCS, FOSM, and PEM (Point Estimate Method), among others [14]. Understandably, each algorithm has its own advantages and disadvantages. For example, although MCS is considered a more reliable method, its enormous computation process makes it impractical when the performance function is too complex. On the other hand, although FOSM and PEM disregarding the information about a variable’s probability distribution are more computationally efficient than MCS, the key limitation in the two algorithms is their accuracy, especially when input variables considered in the probabilistic analysis are highly skewed or asymmetrical [14].

2.1. The FOSM algorithm

First-order second-moment or FOSM is a well-established algorithm for probabilistic analysis, derived on the basis of the Taylor expansion. Therefore, “first-order” implicates that the terms up to the first order of a Taylor expansion are only retained in the calculation; on the other hand, since variance is the second moment of a random variable in statistics, “second-moment” means that FOSM aims to compute the variance of a target variable to quantify the uncertainty of a problem.

As a result, given the performance function denoted as \( Y = f(X_1, s) \), where \( X_1 \) is input random variables, the mean value of \( Y \) (denoted as \( E[Y] \)) can be approximated as follows based on the FOSM algorithm [14]:

\[
E[Y] = f(E[X_1], E[X_2], \ldots, E[X_n])
\]

(1)

Understandably, \( E[X_i] \) is the mean value of \( X_i \), the input data of a probabilistic analysis.

On the other hand, based on the FOSM algorithm, the variance of \( Y \) (denoted as \( V[Y] \)) can be approximated as follows [14]:

\[
V[Y] = \sum_{i=1}^{n} \left( \frac{\partial Y}{\partial X_i} \right)^2 V[X_i] + 2 \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial Y}{\partial X_i} \frac{\partial Y}{\partial X_j} \text{C}[X_i, X_j] \quad \text{for} \ i \neq j
\]

(2)

where \( V[X_i] \) is the variance of \( X_i \), and \( \text{C}[X_i, X_j] \) is the covariance between \( X_i \) and \( X_j \). (Like mean values, both are the input data of the probabilistic analysis.) In addition, \( \frac{\partial Y}{\partial X_i} \) denotes the derivative of \( Y \) against \( X_i \) at the mean value of \( X_i \). Note that when any of two input variables in a probabilistic analysis are considered independent of each other (covariance \( = 0 \)), the variance of \( Y \) can be calculated as follows in a FOSM analysis [14]:

\[
V[Y] = \sum_{i=1}^{n} \left( \frac{\partial Y}{\partial X_i} \right)^2 V[X_i]
\]

(3)

In summary, Eqs. (1)–(3) present the key algorithms of the FOSM probabilistic analysis, which is derived from the Taylor expansion on the performance function \( Y = f(X_1, s) \).

2.2. Monte Carlo simulation or MCS

MCS is considered a more reliable method for performing a probabilistic analysis [14], commonly used in a variety of studies [15–17]. For a performance function \( Y = f(X_1, s) \), the algorithm of MCS is to generate random values of \( X_1 \) based on their probability distribution observed, then substituting them into the function \( Y = f(X_1, s) \) to obtain a random \( Y \) value. With the calculation repeated for a number of times, the mean value, standard deviation, and other statistics of \( Y \) can be estimated based on a series of \( Y \) values generated with MCS [14].

Although the algorithm of MCS is relatively simple, it involves an enormous computation because of the iterative randomization. Therefore, when the performance function \( Y = f(X_1, s) \) is complex, say it takes “one day” to calculate \( Y \) given \( X_1, s \), MCS then becomes less of a practical method to solve the probabilistic analysis, with the “one-day” calculation needed to repeat for a number of times [18].

3. Seismicity around Taiwan and ground motion model

Fig. 1 shows the locations of more than 55,000 main shocks with \( M_t \geq 3.0 \) (local magnitude) around Taiwan since 1900. Note that this earthquake catalog has been studied and used in a few earthquake studies for Taiwan. For example, a statistical study on the data found that the magnitude of major earthquakes around Taiwan should be a random variable following the Gamma distribution or lognormal distribution [19]. Besides, a seismic hazard analysis derived from the seismicity was also reported, a new methodology different from the conventional PSHA and DSHA [7].

The key features of the catalog were summarized as follows. First, the raw data provided by the Central Weather Bureau Taiwan were subjected to a double-link declustering procedure [20] to remove dependent shocks. As a result, the catalog only contains the main shocks around Taiwan since 1900. Second, analyses showed that the \( M_t \geq 3.0 \) data in the catalog are not complete until year 1978, but for \( M_t \geq 5.5 \) earthquakes, the data are complete since 1900 [19].

As any seismic hazard assessment, ground motion models are the performance function of such an analysis. Generally speaking, ground motion models are an empirical relationship characterizing the correlation between earthquake ground motion (e.g., PGA) and earthquake magnitude and source-to-site distance combined. In Taiwan, several ground motion models were developed with earthquake data around the region [21,22], and used in recent seismic hazard assessments [23,21]. In the following seismic hazard analysis, we also adopted a local ground motion model that was frequently used in earthquake analyses for Taiwan [23,21], as follows:

\[ \text{PGA}(g) = -3.25 + 1.075M_w - 1.723 \ln(D) + 0.156\exp(0.624M_w) + \epsilon \]

(4)
where $M_w$ is moment magnitude, $D$ is the source-to-site distance in km, and $\epsilon$ is the model error following the normal distribution with mean $=0$ and standard deviation = 0.577.

Since earthquake magnitudes were in different units adopted by the earthquake catalog (in $M_l$) and the ground motion model (in $M_w$), a conversion relationship is also needed for this study. Similarly, a local conversion model developed with the earthquake data around Taiwan was used in this study [23]:

$$M_l = 4.53 \times \ln(M_w) - 2.09$$ (5)

4. FOSM seismic hazard assessment

As other seismic hazard assessments, only major earthquakes are considered in the analysis, given that small and moderate events are unlikely to cause structure damages. On the other hand, an earthquake, even a large one, that occurs very far from a site is unlikely to cause damage at the site either. As a result, in this study we employed 5.5 $M_w$ and 200 km as the threshold values for defining a major earthquake, following recent seismic hazard assessments for Taiwan [3,7].

From the earthquake catalog, Fig. 2 shows the spatial distribution of 307 major earthquakes ($M_w \geq 5.5$ and $D \leq 200$ km) around the center of Taipei, the most important city in Taiwan. In other words, the annual rate of such an event around Taipei is about 2.8 per year. We then performed statistical analyses on the 307 events. As shown in Fig. 3, the histograms illustrate the distribution of magnitude and distance of the 307 earthquakes. According, the mean magnitude and its standard deviation and skewness (i.e., third moment of a variable) are 6.12, 0.68, and -0.43, respectively. On the other hand, the mean distance and its standard deviation and skewness are 128.95, 38.67, and -0.43, respectively.

With such input data and the ground motion model given in Eq. (4), we then used the FOSM algorithm in Eqs. (1)–(3) to calculate the mean PGA induced by such a major earthquake, as well as its standard deviation. The analysis shows that the mean PGA and its standard deviation are 0.01g and 0.0139g, respectively. That means when a major earthquake with $M_w \geq 5.5$ within 200 km from Taipei occurs, in average the expected ground shaking at the site would be 0.01g in PGA, with standard deviation equal to 0.0139g.

Next, we can calculate the exceedance probability against any of a motion (e.g., $PGA=0.1g, 0.2g \ldots$). The calculation is on the basis that PGA is a random variable following the lognormal distribution induced by such a major earthquake, as shown in previous FOSM seismic hazard analyses [25].

As a result, the exceedance probability against a given motion $y^\ast$ can be computed as follows [24,25]:

$$\text{Pr}(PGA > y^\ast) = \text{Pr}(\ln PGA > \ln y^\ast) = 1 - \phi\left(\frac{\ln y^\ast - \mu_{\ln PGA}}{\sigma_{\ln PGA}}\right)$$ (6)

where $\phi$ denotes the cumulative density function of a standard normal variate (i.e., mean $=0$ and standard deviation $=1$); $\mu_{\ln PGA}$ and $\sigma_{\ln PGA}$ are the mean and standard deviation of $\ln PGA$, which can be calculated based on the following equations, with those of $PGA$ from previous FOSM seismic hazard analyses [25]:

$$\mu_{\ln PGA} = \ln \mu_{PGA} - \frac{\sigma_{\ln PGA}^2}{2}$$ (7)

and

$$\sigma_{\ln PGA}^2 = \ln \left[1 + \left(\frac{\sigma_{PGA}}{\mu_{PGA}}\right)^2\right]$$ (8)

Note that $\mu_{PGA}$ and $\sigma_{PGA}$ are the input data in this computation, for calculating $\mu_{\ln PGA}$ and $\sigma_{\ln PGA}$ in order to compute PGA exceedance probabilities expressed in Eq. (6). With the two equations, $\mu_{\ln PGA}$ and $\sigma_{\ln PGA}$ in this case study were calculated as $-5.133$ and 1.032, given $\mu_{PGA} = 0.01$ and $\sigma_{PGA} = 0.0139$. As a result, when a major earthquake with $M_w \geq 5.5$ within 200 km from Taipei
occurs, for example, there is a 0.02% probability for PGA to exceed 0.23 g at the study site, based on the mean PGA and its standard deviation estimated with the statistics of such major earthquakes occurring in the past 110 years.

Following the framework of PSHA, we then estimated the annual rate of PGA of exceedance, denoted as $\lambda(\text{PGA} > y^*)$, by taking the annual earthquake rate into account [24]:

$$\lambda(\text{PGA} > y^*) = v \times \Pr(\text{PGA} > y^*)$$  \hspace{1cm} (9)

where $v$ is the annual rate of the major earthquake, and as mentioned previously the PGA exceedance probability can be calculated with Eq. (6). According to the seismicity since 1900, in this case study the annual rate of the major earthquake (i.e., $M_w \geq 5.5$ and $D \leq 200$ km) around Taipei was found at 2.8 per year. Therefore, Fig. 4 shows the annual rate of PGA of exceedance up to 0.5g for the study site, based on the FOSM seismic hazard analysis. For example, the annual rate for PGA to exceed the current design value of 0.23g is about 0.0006 per year, corresponding to a return period around 1700 years.

With the same input data and methodology, we carried out another case study for Kaohsiung City in south Taiwan, the second most important city in Taiwan. From the same earthquake catalog shown in Fig. 1, we found that there were a total of 184 major earthquakes ($M_w \geq 5.5$ and $D \leq 200$ km) occurring around the city since 1900, corresponding to an annual rate of 1.67 per year. Fig. 5 shows the magnitude and distance distributions of the 184 earthquakes. For magnitude, the mean value, standard deviation and skewness are 6.21, 0.68, and 1.21, respectively; for distance, they are 137.68, 40.98, and 0.17.

With the earthquake statistics and the same ground motion model (i.e., Eq. (4)), we also used the FOSM algorithm to calculate the mean PGA and its standard deviation for this case study. The result shows that when a major earthquake around the city occurs, the ground shaking in PGA is expected to have a mean value of 0.0098 g, with standard deviation equal to 0.0135 g.

Similarly, considering the earthquake rate $= 1.67$ per year based on the seismicity since 1900, Fig. 6 shows the annual rates of PGA of exceedance up to 0.5 g, following the calculations shown in Eqs. (6)–(9). For example, the annual rate of PGA $> 0.23$ g around the city of Kaohsiung is around 0.0008 per year, corresponding to a return period about 3400 years.

In summary, Table 1 tabulates the FOSM seismic hazard assessments for the two cities in Taiwan, mainly derived from the statistics of major earthquakes recorded in the past 110 years. Like PSHA, the analyses take into account the uncertainties of earthquake magnitude, location, and motion attenuation, but with a different methodology than PSHA and DSHA. The result shows that...
the annual rates for PGA to exceed the current design value (i.e., 0.23 g) in the two cities are no greater than 0.0006 per year, implicating that the current earthquake-resistant designs implemented in the cities should be robust with reasonable conservatism.

5. Discussions

5.1. How reliable the FOSM estimates?

As mentioned previously, unlike MCS, FOSM probabilistic analyses do not fully utilize the statistics of input variables, so that the FOSM estimates might not be very accurate especially when input variables are highly skewed or asymmetrical [14]. However, there is no guidance how to judge whether the FOSM algorithm should be adopted for a specific problem. Therefore, in order to evaluate the accuracy of the FOSM results of the study, we performed MCS on the same problem and made a comparison between the two.

The MCS calculations of the study are summarized as follows. As any MCS, the first step is to generate random input parameters (here are magnitudes, distances, and model errors) based on their probability distribution. For example, for generating a random earthquake magnitude, the histogram shown in Fig. 3a would be based on.

The next step is to substitute the three random parameters into the performance function (i.e., Eq. (4)) to obtain a random PGA motion. With the randomization repeated for a number of trials, the MCS can reliably estimate the statistics of the target variable (here is PGA). Realizing that the sample size is the key to MCS (the larger, the better), we employed a sample size as large as 50,000 in the following analyses.

Fig. 7 shows the 50,000 PGA samples generated with the MCS about the seismic hazard assessment for Taipei. Accordingly, the mean PGA and its standard deviation are 0.0114g and 0.0175g, respectively, which are close to the FOSM estimates in 0.01g and 0.0135g. On the other hand, Fig. 8 shows the Monte Carlo Simulation for Kaohsiung, suggesting that when a major earthquake around the city occurs, the mean PGA and its standard deviation should be around 0.0108g and 0.0161g, respectively, also close to the FOSM estimates in 0.0098g and 0.0135g.

The reason the FOSM and MCS estimates close to each other should be attributed to an overall low-skewness of the three input variables. That is, the skewness of earthquake magnitude and distance (Figs. 3 and 5) is not particularly high in this probabilistic analysis, with the skewness of the model error (ground motion model) equal to zero because it follows the normal distribution that is perfectly symmetrical.

5.2. Robustness of seismic hazard analysis

It has been pointed out that not a seismic hazard assessment is perfect without challenge [26], so that the robustness of a seismic hazard analysis is not related to methodology, but to a transparent and repeatable process [27]. Besides, it must be noted that a complex method (e.g., PSHA) is not necessarily more reliable than a simple one (e.g., DSHA), considering the natural randomness in earthquake not fully understood [26,27]. Therefore, like many others, this FOSM seismic hazard assessment, which is repeatable with the same input data, is a new, scientific reference to the levels of seismic hazard in the two major cities in Taiwan.

6. Conclusions

The region around Taiwan is known for high seismicity, so that earthquake studies such as seismic hazard analysis are valuable to earthquake hazard mitigation in Taiwan. From a different perspective, this study reports a new seismic hazard assessment for two major cities in Taiwan, mainly based on the observed seismicity since 1900, with the use of the well-established FOSM algorithm to solve the probabilistic analysis considering the uncertainties of earthquake magnitude, location, and motion attenuation.
The result shows that the annual rate for earthquake-induced PGA to exceed the current design value (i.e., 0.23g) is relatively low, with it equal to 0.0006 and 0.0003 per year around Taipei and Kaohsiung, respectively. As a result, from the record of the seismicity since 1900, the new seismic hazard assessment from a different perspective than PSHA and DSHA provides a new, scientific reference to the levels of seismic hazard in the two cities, and offers some support to the robustness of the current earthquake-resistant design implemented in Taiwan.

Acknowledgements

We appreciate the valuable comments of the Editor and Reviewers, making this submission much improved in so many aspects. We are also thankful for the kind support from the Central Weather Bureau Taiwan for providing us their earthquake database.

References

[22] Liu PS, Lee CT, Cheng CT, Sung CH. Response spectra attenuation relations for shallow crustal earthquake in Taiwan. Eng Geol 2011;121:150–64.