Soil Dynamics and Earthquake Engineering 75 (2015) 18–26

Contents lists available at ScienceDirect

Soil Dynamics and Earthquake Engineering

journal homepage: www.elsevier.com/locate/soildyn

Bayesian analysis on earthquake magnitude related to an active fault in Taiwan

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ARTICLE INFO

Article history:
Received 17 October 2014
Received in revised form
11 February 2015
Accepted 29 March 2015
Available online 17 April 2015

Keywords:
Earthquake magnitude
Bayesian approach
Limited observation
Prior data

ABSTRACT

It is understood that sample size could be an issue in earthquake statistical studies, causing the best estimate being too deterministic or less representative derived from limited statistics from observation. Like many Bayesian analyses and estimates, this study shows another novel application of the Bayesian approach to earthquake engineering, using prior data to help compensate the limited observation for the target problem to estimate the magnitude of the recurring Meishan earthquake in central Taiwan. With the Bayesian algorithms developed, the Bayesian analysis suggests that the next major event induced by the Meishan fault in central Taiwan should be in M_w 6.44 $\pm$ 0.33, based on one magnitude observation of M_w 6.4 from the last event, along with the prior data including fault length of 14 km, rupture width of 15 km, rupture area of 216 km^2, average displacement of 0.7 m, slip rate of 6 mm/yr, and five earthquake empirical models.

1. Introduction

The region around Taiwan is known for high seismicity, not to mention a catastrophic event like the M_w 7.6 Chi–Chi earthquake could recur in decades [1]. Recently, there are studies suggesting that the return period of a major earthquake induced by the Meishan fault in central Taiwan might be as short as 160 years, not to mention the very last Meishan earthquake in 1906 was occurring more than one hundred years ago. Under the circumstances, the risk of the active fault inducing a major earthquake in near future is considered relatively high, and the subject has been discussed in several recent studies [2,3]. Therefore, from a different perspective with new methodology, the target problem of this study is to evaluate the magnitude of the next Meishan earthquake in central Taiwan that could occur in near future given its short return period. More introductions to the background of the Meishan fault in central Taiwan are given in one of the following sections.

One possibility to evaluate such a problem is via statistical study. But on the other hand, it is understood that sample size is important to statistical assessments and inferences. For example, given an active fault is known for inducing a major earthquake in M_w 6.5 (moment magnitude), a best estimate on the magnitude of the next recurrence will be exactly the same size as the observation, although it is less representative based on one magnitude observation only. Unfortunately, this is the same situation for the target problem of the study, aiming to evaluate the magnitude of the next Meishan earthquake but with only one magnitude observation available for the analysis.

In contrast to statistics-based methods, the Bayesian inference is a relatively new approach that is more useful for evaluating a problem with very limited observations. Basically, the Bayesian approach is to use other sources of data to compensate limited statistics, helping develop a new Bayesian estimate by integrating multiple sources/types of data, usually referred to as prior and observation.

The Bayesian approach has been increasingly applied to many different studies to develop a new estimate from multiple sources of data e.g., [4–6]. In earthquake engineering and engineering seismology, an early study can be dated back to the 1960s [7], introducing the framework of the Bayesian calculation for seismology research. More recently, several other Bayesian methods for earthquake studies were reported, such as the application to earthquake early warning [8,9], tectonic stress evaluation [10], and earthquake catalog characterizations [11], among others [12,14].

Although many different applications were reported, the underlying motivation of the Bayesian studies is the same: Integrating multiple sources/types of data to evaluate or re-evaluate a problem, rather than only relying on (limited) statistics from observation. Take

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http://dx.doi.org/10.1016/j.soildyn.2015.03.025
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those studies above for example, the framework of Bayesian earthquake early warning is to utilize some empirical models to compensate the limited data at the initial stage of earthquake, for estimating its magnitude and location more reliably on a real-time basis [8,9]. On the other hand, a Bayesian algorithm [11] to evaluate the completeness magnitude (M_c) of an earthquake catalog is facilitated with the prior data in proximity regions, then integrated with the locally observed seismicity. Note that such a Bayesian calculation is similar to a later application to estimate earthquake rates (or frequencies) around a study area, also using the data from proximity areas as priors [14]. Other recent Bayesian applications to earthquake studies include the probability assessment on earthquake-induced landslides [31], evaluation of the source parameters of a major earthquake [32], and structure safety analysis under earthquake condition [33]. Similarly, new Bayesian methods are increasingly developed for other problems [34–36].

As a result, given the short return period reported, the key scope of the study is to evaluate the magnitude of the next major earthquake induced by the Meishan fault in central Taiwan, on the basis of a novel Bayesian calculation integrating multiple sources/types of data to compensate the lack of adequate statistics from observation. In this study, we first derived a new Bayesian algorithm for evaluating earthquake magnitude distributions related to an active fault, based on both observational and prior data. Next, we applied the methodology to the target problem, showing there should be a 10% probability for the next Meishan earthquake in central Taiwan to exceed M_w 6.9, considering one magnitude observation of M_w 6.4 from the last Meishan earthquake, and the prior data including fault length of 14 km, rupture width of 15 km, rupture area of 216 km^2, average displacement of 0.7 m, slip rate of 6 mm/yr, and five earthquake empirical models.

The paper is organized with an overview of the Bayesian approach, followed by the introductions to the Meishan fault in central Taiwan. Next, the observation and prior data for this Bayesian study were introduced and summarized, followed by the developments of the new Bayesian algorithm, and the Bayesian inference to the magnitude of the next Meishan earthquake from the multiple sources/types of data.

2. Overview of the Bayesian approach

2.1. The algorithm

As mentioned previously, the Bayesian approach is to integrate prior information with (limited) observation to develop a new estimate, which is different from the one relying on samples or statistics only. To further illustrate the method, we summarized an example from the literature as follows [15]: Fig. 1a shows the prior information or the so-called prior probability mass function about the accident at a given cross road, suggesting the mean rate equal to two accidents per year. It is worth noting that because this example is a discrete case, its probability function is specifically referred to as probability mass function (PMF), in contrast to probability density function (PDF) that is used for describing the probability function of a continuous random variable [15]. Nevertheless, the two basically refer to the same thing in probability and statistics.

On the other hand, given the total number of accidents equal to one observed in an arbitrary month, the accident rate should be 12 per year from the observation. Note that although the reference only mentions "one accident observed in one month" in the description to the example [15], the description should explicitly imply the one-month observation was conducted in an arbitrary period of 30 (or 31) days in a row. Therefore, in this paper we refer to such a description or the observation as "one-accident-in-one-arbitrary-month" in the following.

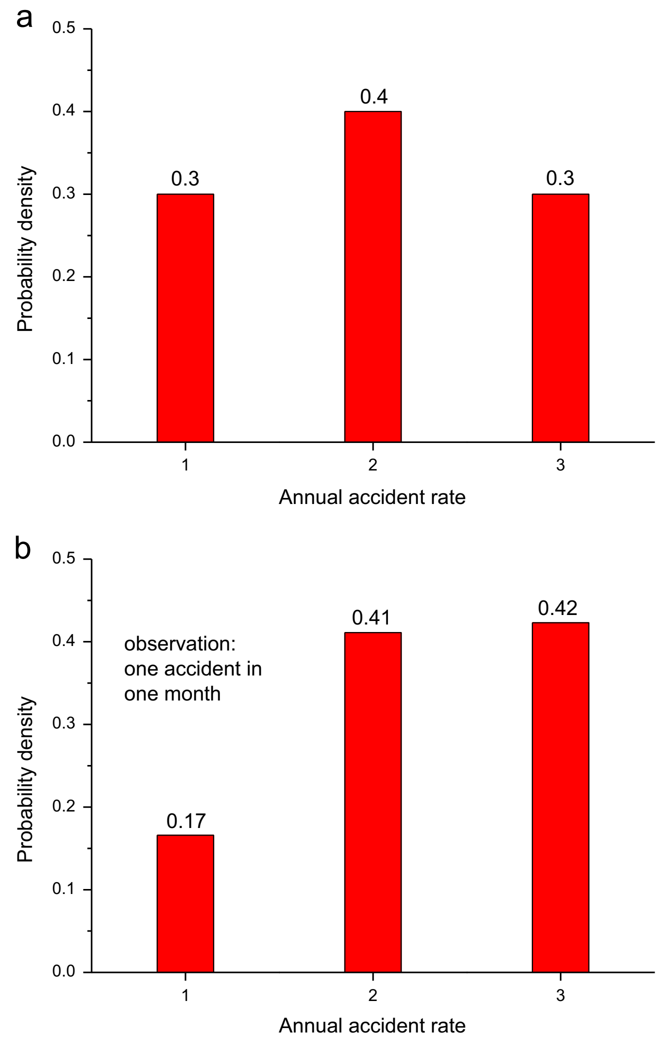

Fig. 1. A demonstration example to the Bayesian approach: (a) the prior information or the prior probability mass function, and (b) the posterior integrating the prior with the observation [1].

In addition to the two estimates from observation or prior, the third one is the Bayesian estimate by integrating the two. For a discrete case like this demonstration example, the underlying algorithm of the Bayesian approach can be expressed as follows [15]:

$$P''(\theta_i) = \frac{P'(\theta_i) \times P(\varepsilon | \theta_i)}{\sum_{i=1}^{n} P'(\theta_i) \times P(\varepsilon | \theta_i)} \quad (1)$$

where ε denotes observation, $P'(\theta_i)$and$P''(\theta_i)$are prior and posterior probabilities for each prior estimate θ_i, and $P(\varepsilon | \theta_i)$ is the likelihood function, or the probability for observation ε to occur given θ_i.

Understandably, θ_i, $P'(\theta_i)$, and ε are the given data in a Bayesian calculation. (For this demonstration example, θ_i are 1 or 2 or 3 accidents, $P'(\theta_i)$ are 30% or 40% or 30%, and ε is the "one-accident-in-one-arbitrary-month" observation.) By contrast, $P(\varepsilon | \theta_i)$ and $P''(\theta_i)$ are unknowns that we want to calculate during the Bayesian analysis. More importantly, from the unique algorithm given in Eq. (1), we can see how the Bayesian approach integrates prior data and observation with the well-established algorithm.

It is worth noting that in the calculation of the likelihood function $P(\varepsilon | \theta_i)$, we need to know (or assume) what kind of probability distributions the target random variable should be following. That is, in this demonstration example, the accident rate is considered following the Poisson distribution, with its probability mass function defined as follows [15]:

$$P(X=x;\nu)=\frac{e^{-\nu}\nu^{x}}{x!} \tag{2}$$

where ν is the mean value of a Poissonian (discrete) random variable X.

With the Poissonian presumption, therefore the likelihood function for the prior estimate $\theta=1$ of this example can be calculated as follows:

$$P(\varepsilon=\text{one accident in one arbitrary month}|\theta=1\text{ accident per year})$$

$$=\frac{e^{-1/12}(1/12)^{1}}{1!}=0.077 \tag{3}$$

As a result, the probability or the likelihood function for such an observation to occur given the prior estimate $\theta=1$ is equal to 0.077. It is worth noting that the Poisson calculation above involved a transformation of the model parameter (which is mean value) from $\theta=1$ per year to $\theta=1/12$ per month, which is a necessary step in the calculation when units are inconsistent. However, for this specific Bayesian calculation, we cannot transform the given observation from "one-accident-in-one-arbitrary-month" to "12-accident-in-one-arbitrary-year," because the two are different observations in essence, with the one-year observation being a bigger sample size than the one-month observation.

By repeating such calculation for another two estimates, three respective likelihood functions are then available. Together with the three prior probabilities given, the posterior probabilities for each estimate can be calculated with the Bayesian algorithm (i.e., Eq. (1)), and the so-called posterior function shown in Fig. 1b can be developed.

The final step of a Bayesian analysis is to develop an estimate based on the updated, posterior information. Therefore, in this demonstration example, the Bayesian estimate is equal to 2.26 accidents per year based on the posterior information shown in Fig. 1b. In other words, the Bayesian estimate is a result of the "one-accident-in-one-arbitrary-month" observation, and the prior information shown in Fig. 1a.

2.2. The principle of the Bayesian updating

From the updated, posterior information (Fig. 1b), we can see that given the "one-accident-in-one-arbitrary-month" observation, the (posterior) probability for the prior estimate of $\theta=3$ (three accidents per year) increases to 42%, while it decreases to 17% for the prior estimate of $\theta=1$. Therefore, the principle of the Bayesian updating can be summarized as follows: When a prior estimate is relatively close to the observation, its probability will increase after updating; otherwise it will decrease. That is, because the prior estimate of $\theta=3$ is closer to the observation (suggesting 12 accidents per year) in this demonstration example, its (posterior) probability must be increasing after the updating from the observation.

3. Prior information regarding the Meishan fault in central Taiwan

3.1. Overview of the Meishan fault

The region around Taiwan located in the boundary of three tectonic plates is known for high seismicity. Especially after the infamous M_w 7.6 Chi–Chi earthquake in 1999, the Central Geological Survey Taiwan launched a comprehensive investigation on active faults in Taiwan [16,17], and periodically updated the findings from the research. For example, the Central Geological Survey Taiwan reported their best-estimate return period as 160 years for a major earthquake (like the Meishan earthquake in 1906) induced by the Meishan fault in central Taiwan. More importantly, considering the very last M_w 6.4 event occurring in 1906, the active fault should be "dangerous" to the surrounding areas given the return period is "almost" due [2,3].

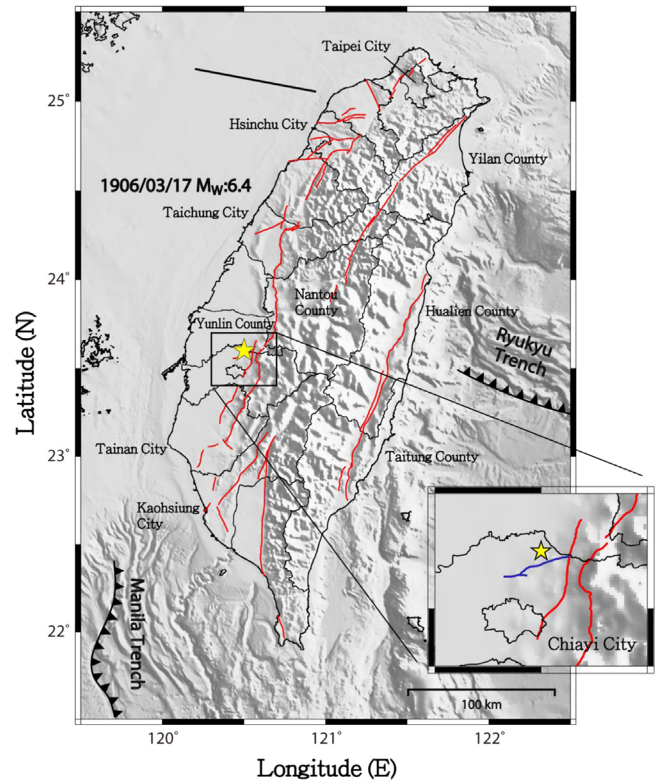

Fig. 2. The location of the Meishan fault in central Taiwan, and the epicenter of the 1906 M_w 6.4 Meishan earthquake.

Fig. 2 shows the location of the Meishan fault in central Taiwan, and the epicenter of the M_w 6.4 Meishan earthquake in 1906 that killed around 3,000 people at that time [18]. (Given the earthquake epicenter in Meishan Township, the fault and the earthquake were then referred to as the Meishan fault and the Meishan earthquake.) Preliminary field investigations reported the fault was a strike-slip fault, extending in an almost east-west direction with the fault length in 10–15 km [19]. Recently, some studies from modern instrumentation suggested that the rupture zone might be more complex and extensive than earlier reports [20].

Because of the background of the active fault, some recent studies focusing on the earthquake probability and the monitoring of the fault were reported. For example, a study suggested that there should be a 30% probability for the next Meishan earthquake to recur in next 50 years, with a new non-stationary model considering the 160-year return period and the last occurrence in 1906 [2]. In addition, another study suggested that there should be a 2% probability for the Meishan fault to induce a catastrophic earthquake like the M_w 7.6 Chi–Chi earthquake, from a novel application of advance first-order second-moment (AFOSM) probabilistic analysis [18]. As for field monitoring, an intensive GPS network was installed along the fault to "watch" its activity on a daily basis [3].

3.2. The "properties" of the Meishan fault

As mentioned previously, because there is only one magnitude observation, we would like to collect some prior data for the target problem to estimate the magnitude of the next Meishan earthquake in central Taiwan. In a recent seismic hazard study for Taiwan [21], we found the "properties" of the Meishan fault reported as follows: fault length of 14 km, rupture width of 15 km, rupture area of

216 km^2, average displacement of 0.7 m, and slip rate of 6 mm per year. It is worth noting that although the study did not explicitly state where the data came from, they should be a result of in-house investigations and analyses, from field reconnaissance, to fault trenching, to GPS data, etc [21].

3.3. Empirical relationships between earthquake magnitude and active faults

With the "properties" of the Meishan fault available, a following task of the study is to search for some empirical models that can predict earthquake magnitudes from the fault data. From the literature, we found five empirical relationships for the purpose, from the studies of Wells and Coppersmith, Anderson et al. [22,23]:

$$M_w = 5.08 + 1.16 \log L \pm 0.28 \quad (4)$$

$$M_w = 4.06 + 2.25 \log W \pm 0.41 \quad (5)$$

$$M_w = 4.07 + 0.98 \log A \pm 0.24 \quad (6)$$

$$M_w = 6.93 + 0.82 \log D \pm 0.39 \quad (7)$$

$$M_w = 5.12 + 1.16 \log L - 0.2 \log S \pm 0.23 \quad (8)$$

where L, W, A, D, and S denote fault length (in km), rupture width (in km), rupture area (in km^2), average displacement (in m), and slip rate (in mm/yr), respectively. Also note that those "$\pm$" terms in the equations are standard deviation, or the standard deviation of model error of the regression models.

With such regression models, then the magnitude of major earthquakes related to an active fault can be estimated from fault data. For example, given fault length = 10 km, the mean earthquake magnitude associated with the fault is about M_w 6.2 from Eq. (4), with a standard deviation of 0.28.

3.4. Reviews of regression analysis

Here, we would also like to elaborate more about regression analysis that is part of the Bayesian analysis and estimate. By definition, a regression model can be expressed as follows [15]:

$$Y = aX + b + e \quad (9)$$

where X and Y are the so-called independent and dependent variables, a and b are model parameters (constants), and e is model error, a random variable following the normal distribution with mean value = 0 by definition [15]. (Note that in order to avoid confusion from the Bayesian algorithm (Eq. (1)), here we denoted model error as e instead of ε that is usually adopted in regression analysis.)

Therefore, given $X = 10$ for example, Y is then equal to $(a \times 10 + b + e)$, where $(a \times 10 + b)$ is obviously a constant, and e is still a random variable. As a result, if e is a variable following the normal distribution by definition, Y given $X = 10$ (or any given values) will be the same type of variables. We used an analogy to further explain the simple relationship as follows: given $C = D + 1$, the two random variables will follow the same probability distribution regardless.

Understanding the fundamentals of regression models, earthquake magnitude (as Y in Eq. (9)) estimated from regression relationships will be a random variable following the normal distribution, given the model error of any regression relationships follows the normal distribution [15]. Therefore, considering regression models are part of the Bayesian calculation, the magnitude of recurring major earthquakes following the normal distribution is a mathematically robust presumption from the theory of probability and statistics.

On the other hand, to the best of our knowledge, due to the lack of adequate statistics, there is no statistical study available that provides tangible evidence about the magnitude distribution of recurring major earthquakes induced by a given fault. Even from the well-known Gutenberg–Richter recurrence law [24], the relationship is simply about a strong correlation between the logarithm of earthquake numbers and the magnitude of exceedance in regional seismicity, but without any indications to what probability distributions (neither normal nor lognormal) the magnitude of major earthquakes induced by the same active fault should be following.

3.5. Independence of data

Understandably, the fault data and empirical models summarized above would be used as prior data in this Bayesian study, in contrast to the magnitude (M_w 6.4) observation from the last Meishan earthquake. But before any Bayesian calculations, it is imperative to justify the two are from independent sources of data; otherwise the Bayesian analysis will be in question. That is, if the prior is derived from observation or in the other way around, the two are basically from the same source of data, outside the scope of the Bayesian analysis using multiple data that were developed independently.

Therefore, in this section we would like to summarize our justifications to the data independence of this Bayesian study. As mentioned previously, the first evidence was from the study where the fault data (e.g., fault length, slip rate) were reported, implying they were a result of in-house investigations, independent of the other two sources of data used in this study [21]. Second, we looked up the raw data given in the papers where the five empirical models were originated [22,23], and found that the data of the Meishan fault were not included in the database. Finally, we cross checked the magnitude observation (M_w 6.4) and the five magnitude estimates from the prior data, and found that they are all different from each other (see Fig. 3). Understandably, this evidence also provides some justification to the data independence; otherwise the magnitudes must be identical if they are back-calculated from each other.

4. Estimating earthquake magnitude with the empirical models

Before introducing the new Bayesian analysis, two conventional estimates based on the fault data and empirical relationships are given in the following: (1) the deterministic approach neglecting model error, and (2) the probabilistic approach considering model error. Understandably, the two estimates are irrelevant to the statistics from observation.

4.1. Deterministic estimate

Without considering the model error of the regressions, Fig. 3 shows the five estimates from this so-called deterministic approach. For example, given fault length = 14 km, the deterministic estimate is equal to M_w 6.41 from Eq. (4), irrelevant to the model error of ± 0.28 of this empirical relationship. In other words, the deterministic assessment did not consider the model error in the analysis.

The deterministic approach shows that the highest estimate of M_w 6.8 among the five scenarios is from Eq. (7) based on the fault displacement of 0.7 m; by contrast, using fault length and slip rate combined ($L = 14$ km, slip rate $S = 6$ mm per year, and Eq. (8)) returns the lowest estimate of M_w 6.29 among the five.

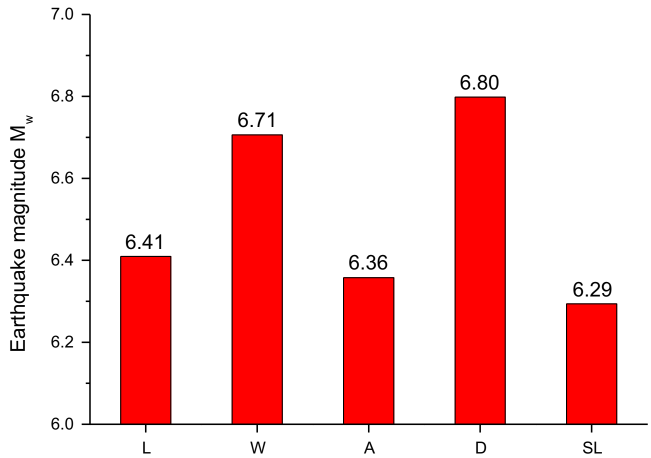

Fig. 3. The five deterministic estimates from the fault data and earthquake empirical models, where "*L*" denotes fault length, "*W*" for rupture width, "*A*" for rupture area, "*D*" for average displacement, and "SL" for slip rate and fault length combined.

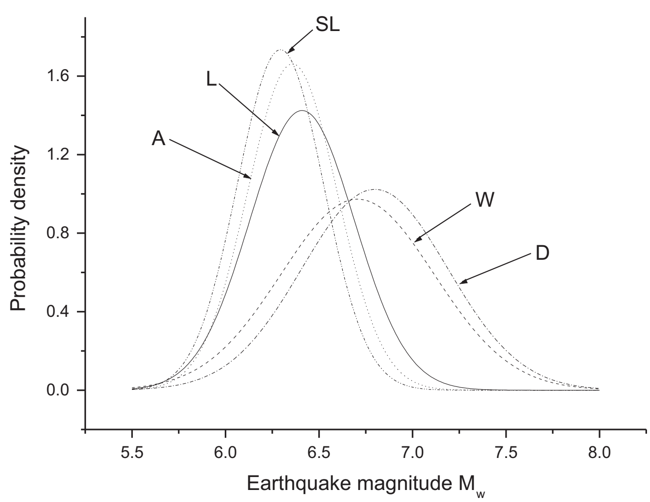

Fig. 4. The five probabilistic estimates considering the uncertainty of the earthquake empirical models.

4.2. Probabilistic estimate

Fig. 4 shows the five probabilistic estimates with model error taken into account. For example, the estimate of M_w 6.71 $\pm$ 0.41 based on rupture width of 15 km suggests a broader probability distribution, given the larger error of $\pm$ 0.41 in Eq. (5). By contrast, the estimate with the smallest variability among the five is M_w 6.29 $\pm$ 0.23 from Eq. (8), based on fault length of 14 km and slip rate of 6 mm/yr combined.

As elaborated in Section 3.4, the five random variables in/from regression models follow a bell-shaped normal distribution, given the model error of regression following the normal distribution by definition [15]. Also note that it is correct in Fig. 4 where the probability density (not cumulative probability) is greater than 1.0, because earthquake magnitude is a continuous random variable.

5. The Bayesian calculation and estimate

5.1. The algorithms

In contrast to the two estimates above, this section would like to show how to apply the Bayesian approach to merge the limited observation with the prior data in this study, developing a new estimate for the target problem given limited statistics from observation are only available.

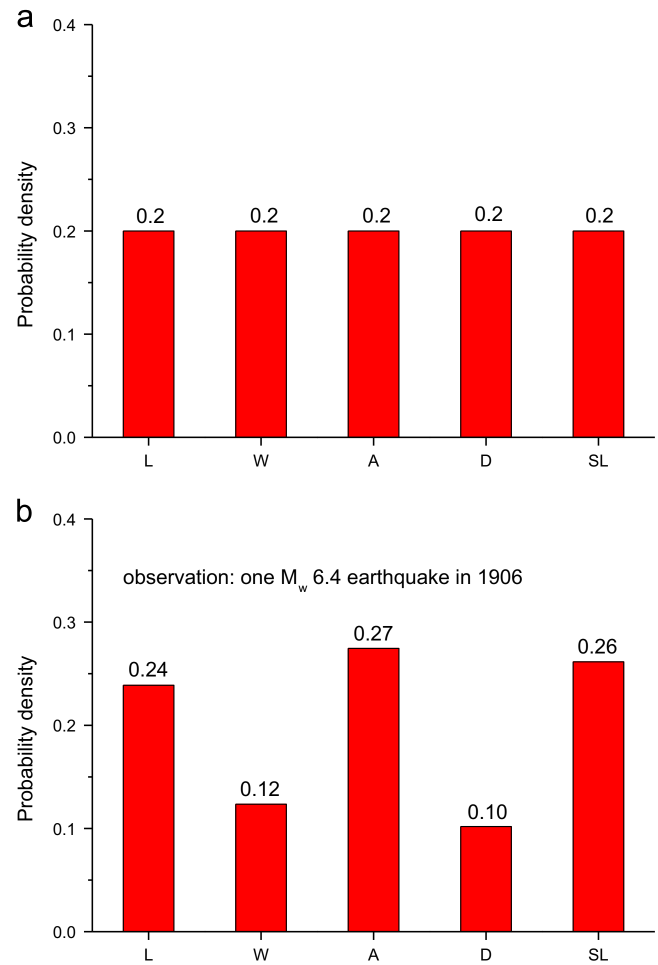

Fig. 5. The Bayesian analysis for this study: (a) the "diffuse" prior with an equal probability assigned to each of the five scenarios, and (b) the posterior integrating the prior with one earthquake (M_w 6.4) observation in 1906.

As many Bayesian studies e.g., [4,6,14], we also assigned a "diffuse" prior to each of the five scenarios; that is, without further support, we considered each scenario has an equal probability, leading to a 20% prior probability for each of them. As a result, Fig. 5a shows the prior probability function for the Bayesian analysis, where "*L*" denotes the estimate from fault length, and so on.

The next step, probably the most challenging one as in any Bayesian methods, is the calculation of the likelihood function $P(\varepsilon|\theta_i)$. Like the demonstration example, we need to know what type of probability distributions earthquake magnitude is following before the calculation. As explained previously (mainly given in Section 3.4), we used the normal distribution for the calculation of the likelihood function, which is a rational, defensible presumption in this probabilistic study, not to mention it was also used in many probabilistic analyses especially when no tangible evidence is available [29]. (For example, in geotechnical engineering the factor of safety is usually considered a random variable following the normal distribution, which is our best engineering presumption rather than a finding from statistical studies [29].)

As a result, the likelihood function of this Bayesian study can be derived as follows:

$$P(\varepsilon|\theta_i) = \prod_j g(x_j;\mu_i,\sigma_i) = \prod_j \frac{1}{\sqrt{2\pi}\sigma_i}\exp\left(\frac{-(x_j-\mu_i)^2}{2\sigma_i^2}\right) \tag{10}$$

where θ_i denotes each of the five scenarios, μ_i and σ_i are the mean magnitude and standard deviation of each scenario (see Fig. 4), x_1, …,x_j denote each sample in observation ε, and g denotes the normal distribution, with its probability density function also shown in Eq. (10) [15]. Note that like any Bayesian algorithms, the formula is a general expression, meaning it is applicable to an observation consisting of one sample, two samples, etc.

With the likelihood functions available, the rest of the calculations are relatively straightforward by combining Eqs. (1) and (10). Therefore, the posterior probability of this Bayesian study can be calculated with Eq. (11), integrating observation with prior data by such a unique manner:

$$P''(\theta_i) = P'(\theta_i) \times \frac{\prod_j \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left(\frac{-(x_j-\mu_i)^2}{2\sigma_i^2}\right)}{\sum_{i=1}^{n} P'(\theta_i) \times \prod_j \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left(\frac{-(x_j-\mu_i)^2}{2\sigma_i^2}\right)} \quad (11)$$

As mentioned previously, the prior probabilities$P'(\theta_i)$ are all equal to 20% in this Bayesian study, given no further evidence that can quantify relative reliability among the five scenarios.

5.2. The updating and the estimate

From data collections to Bayesian analyses, Fig. 5b shows the posterior probability for each scenario based on the observation and prior. Accordingly, the probability increases to 27% for the rupture-area scenario, because the M_w 6.4 observation from the last Meishan earthquake is relatively similar to the estimate of M_w 6.36 ± 0.24. By contrast, the probability for the rupture-width scenario decreases to 10%, given its estimate of 6.8 ± 0.39 is relatively different from the same observation. With the same updating for the rest of the scenarios, Fig. 5b shows the final posterior probability function of the Bayesian analysis.

For better explaining and understanding the Bayesian updating, Fig. 6 shows the likelihood function for each of the five scenarios. From the calculations, we can clearly see the likelihood functions are indeed proportional to posterior probabilities as shown in Fig. 5b; to be more specific, the larger the likelihood function, the larger the posterior probability will be after the updating from observation. As mentioned previously, it is correct in Fig. 6 where the probability density (not cumulative probability) is greater than 1.0, because earthquake magnitude is a continuous type of random variables.

5.3. The Bayesian estimate

Like other Bayesian studies, the final step of this Bayesian analysis is to develop a Bayesian estimate based on the posterior information shown in Fig. 5b. Accordingly, Fig. 7a shows the magnitude distribution for the next Meishan earthquake, with mean value = M_w 6.44 and standard deviation = 0.33. Based on the probability function, we then developed the relationship between exceedance probability and earthquake magnitude, suggesting there should be a 50% probability for the next Meishan earthquake to exceed M_w 6.4, or a 10% probability to exceed M_w 6.9, and so on.

Specifically, we used Monte Carlo Simulation (MCS) with a very large sample size of 100,000 to develop the Bayesian estimate. The details about the MCS analysis are given in the Appendix.

The reason of using MCS rather than simple linear combination [15] for the analysis can be explained with a schematic diagram shown in Fig. 8. Let's say a variable X follows a "0 ± 1" or "10 ± 1" normal distribution in 50-to-50 probability, and owing to the uncertainties, we can expect that the final estimate for the variable should be associated with a larger variability than the two basic scenarios. However, when applying simple linear combination to the analysis, the standard deviation was then

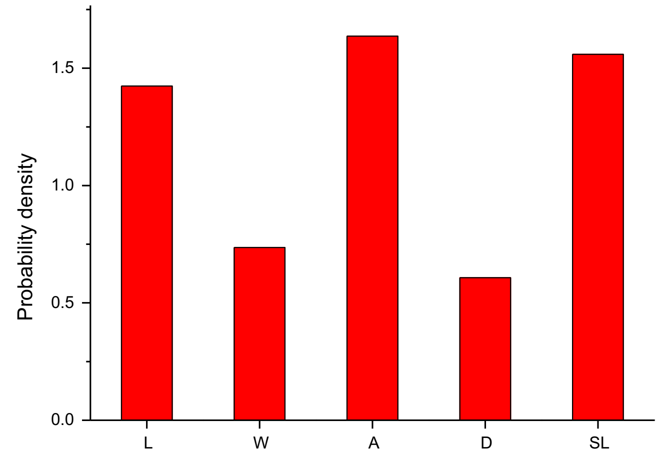

Fig. 6. The likelihood functions for each of the five scenarios subject to one magnitude observation of M_w 6.4.

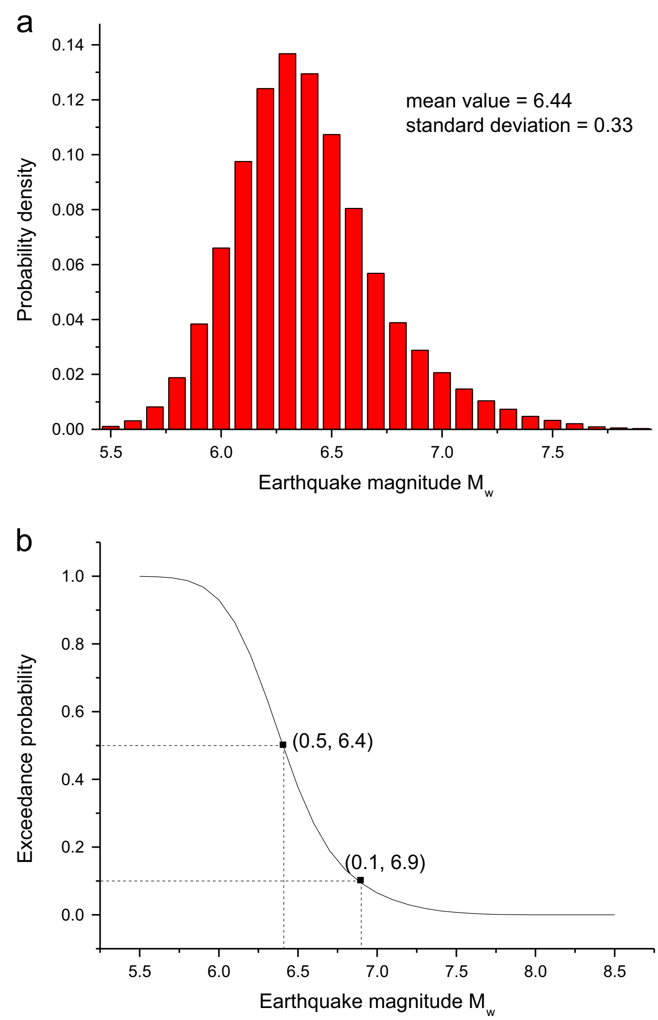

Fig. 7. The Bayesian analysis estimating the magnitude of the very next Meishan earthquake based on the posterior information in Fig. 5b: (a) the probability density function with mean = 6.44 and standard deviation = 0.33, and (b) the relationship between exceedance probability and earthquake magnitude.

equal to $\sqrt{0.5^2 \times 1^2 + 0.5^2 \times 1^2} = 0.71$ based on the algorithm [15], which is smaller than that of the two given priors.

On the other hand, Fig. 8 also shows the MCS estimate for the analysis, which suggests a much more reasonable distribution for X subject to the priors. In short, the reason for linear-combination

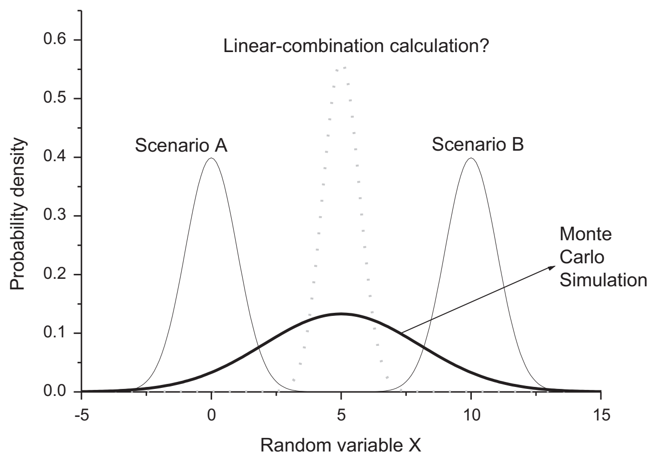

Fig. 8. A schematic diagram showing two estimates for the variable's probability function with Monte Carlo Simulation and linear-combination algorithms.

algorithms not applicable to this study is mainly because prior or posterior probabilities assigned to each scenario must be less than 1.0 in such a discrete case of the Bayesian analysis, leading to a smaller standard deviation on the basis of the linear-combination algorithm of random variables.

6. Discussions

6.1. Robustness of the methodology

Understandably, the Bayesian algorithm of this study from the well-established Bayesian approach is by all means transparent and robust, like those Bayesian studies summarized in the paper. For example, the Bayesian calculations for earthquake catalog characterizations are applicable to any databases, although the method was only applied to the seismicity around Taiwan as a demonstration in that paper [11]. Similarly, another Bayesian application to the calculations of "*b*-value" (i.e., a parameter of the Gutenberg–Richter model) can be used for any other regions, although the authors of the paper only applied it to the earthquake data around Japan as a case study [13]. Therefore, this study is in the same situation: the new Bayesian algorithm derived from the well-established approach is completely robust, although we only applied it to our target problem.

Given the methodology is mathematically robust, it should become redundant to conduct more case studies just for justification, given the case studies are of less interest and importance than a target problem. As a result, this is the reason why we only selected the case in central Taiwan accompanying the new methodology developed, which is the target problem and the motivation of the study to estimate the magnitude of a major earthquake with a short return period.

On the other hand, in many earthquake studies it is common to see a "local" case study usually accompanies new methodology developed, mainly owing to the expertise/experience of researchers, and the availability of reliable data available to them. That is, it is common that American researchers use earthquake data in California as some support to new methodology and theory [24,25], while Japanese and Taiwanese researchers use the data around Japan and Taiwan in their works [26,27], and so on [28]. Similarly, the situation can be seen in those Bayesian studies mentioned earlier [8–14].

Nevertheless, in order to further attest the new, robust Bayesian method is applicable to any similar cases, we used some hypothetical data to evaluate the earthquake magnitude of an active fault with the data as follows: rupture length = 30 km, rupture width = 25 km, rupture area = 800 km^2, average displacement = 1 m, slip rate = 5 mm/yr, along with three magnitude observations of M_w 7.0, 7.2, and 7.3 from the past. With the data and the Bayesian calculation, Fig. 9a shows the prior and posterior probabilities for the analysis, indicating the probability of the rupture-width scenario increases to 38%, given the observations of M_w 7.0, 7.2, and 7.3 are in more support of this magnitude estimate of M_w 7.2. By contrast, the probability of the slip-rate scenario would decrease to less than 1%, because its estimate of M_w 6.69 from Eq. (8) was very different from the three observations of 7.0, 7.2, and 7.3. Similarly, based on the updated, posterior information, Fig. 9b shows the magnitude distribution with the same MCS procedure, suggesting there should be, for example, a 10% probability for the fault to induce an earthquake above M_w 7.5, given the group of information including L = 30 km, W = 25 km, A = 800 km^2, D = 1 m, S = 5 mm/yr, five empirical models, and three magnitude observations of M_w 7.0, 7.2, and 7.3.

6.2. On the proper use of Bayesian inferences

Clearly, the Bayesian approach is a well-established algorithm that has been increasingly applied to a variety of studies, especially when observations are rather limited. In other words, the method is to use prior data to compensate limited samples in order to develop a more "accountable" estimate. However, it must be noted that the Bayesian estimate is just another best estimate like any others. In order to prove, mathematically, one estimate is better

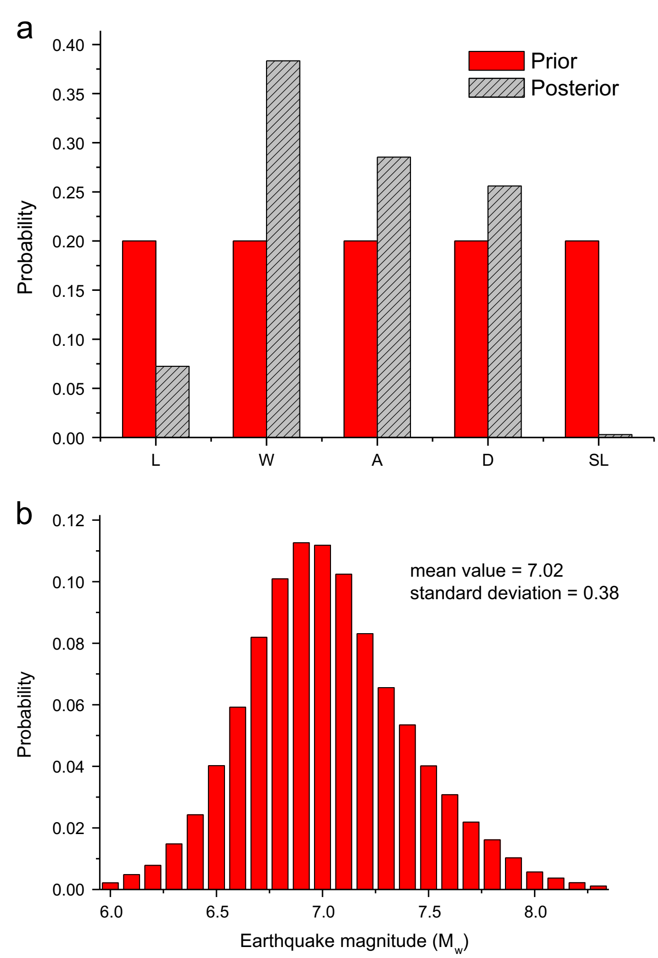

Fig. 9. The additional application to a similar example like the target problem of the study: (a) the prior and posterior functions, and (b) the Bayesian estimate on the magnitude distribution based on the posterior information, from the data including L = 30 km, W = 25 km, A = 800 km^2, D = 1 m, S = 5 mm/yr, and the three observations of M_w 7.0, 7.2, and 7.3.

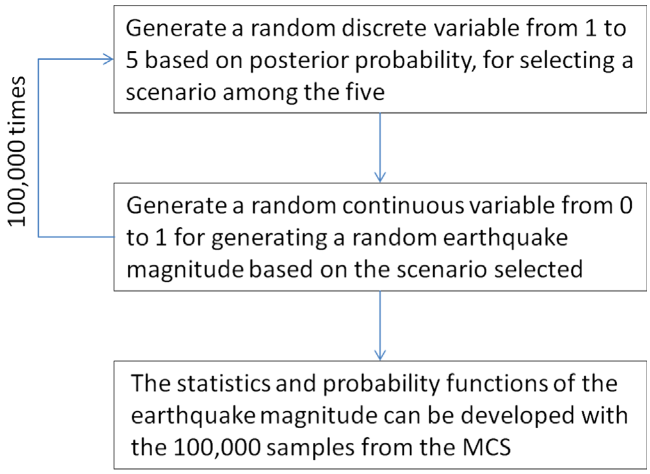

Fig. A.1. The flowchart of the MCS procedure to develop the Bayesian estimate based on the posterior information.

than the other, the calculations of their "consistency," or "efficiency," or "sufficiency" are in need for the evaluation [15]. But because the Bayesian approach is fundamentally different than classic statistics and probability, those calculations for evaluating a Bayesian estimate seem "impossible," and this might be the reason why such calculations and evaluations were not reported in other Bayesian studies either [4–6,8–14].

Nevertheless, we consider this Bayesian estimate (i.e., 6.44 ± 0.33) developed with both (limited) observation and prior data should be more representative than the one derived from one magnitude observation. But interestingly, in this Bayesian study the two estimates are close to each other in terms of mean values (6.44 vs 6.4), suggesting that the prior data and the observation from independent sources might be somehow comparable. On the other hand, the Bayesian approach is an alternative for evaluating the variability of the target problem subject to inevitable nature randomness, which cannot be obtained with one sample, or the variability is zero.

6.3. Earthquake randomness

Understandably, earthquake randomness could appear in magnitude (the focus of the study), as well as in location, recurrence date, etc. Clearly, earthquake randomness in magnitude and in recurrence date is different problems in essence, and they must be analyzed with different analyses and approaches. For example, the temporal earthquake uncertainty is usually modeled by the Poisson distribution [28,30]; by contrast, the uncertainty of earthquake magnitude can be analyzed from the empirical models [22,23], as well as the Bayesian method of the paper.

However, the two probability estimates on different problems can be easily integrated. Take the target problem of this study for example, given the return period of the Meishan earthquake of 160 years, the commonly used Poisson calculation would suggest a 8% probability for the event to recur in next 50 years. As a result, by integrating this estimate with the result of this study, the probability for the Meishan fault to induce a major earthquake above M_w 6.9 in the next 50 years should be equal to 0.08 (time uncertainty) $\times$ 0.1 (size uncertainty) = 0.008, a simply example demonstrating the two different problems and the integration.

7. Summary and conclusion

Taiwan is known for high seismicity; in particular, the Meishan fault in central Taiwan with a relative short earthquake return period should be "dangerous" to the surrounding areas. As a result, this study is aimed at estimating the magnitude of the Meishan earthquake that could recur in near future, given a best-estimate return period of the event as short as 160 years.

However, with only one magnitude observation from the last Meishan earthquake in 1906, the estimate is not representative based on the limited statistics. Therefore, as many Bayesian studies using prior data to help compensate limited data from observation, this study presents a new Bayesian application to the target problem to evaluate the magnitude of the next Meishan earthquake, based on one magnitude observation, along with the prior data including the "properties" of the Meishan fault and earthquake empirical models from independent sources.

With the prior data and the limited observation, the new Bayesian study suggests that the magnitude of the very next Meishan earthquake could be in M_w 6.44 ± 0.33; or there should be a 50% probability for the earthquake to exceed M_w 6.4, or a 10% probability to exceed M_w 6.9, and so on. Nevertheless, the Bayesian estimate from multiple sources of data should be more representative than the one relying on one magnitude observation for this target problem.

Acknowledgements

We appreciate the comments and suggestions from Editor and reviewers making this submission much improved in so many aspects. The author would also like to thank the Hong Kong University of Science and Technology for the financial support on the study (Grant FSGRF12EG49).

Appendix. Developing probability functions with MCS

The flowchart shown in Fig. A1 summarizes the MCS procedure to generate random samples based on the given (posterior) data, and to develop the earthquake magnitude's statistics and probability functions. The simulation was assisted with the random number generator in Excel (function RAND) along with common randomization algorithms such as the inversed method [29].

References

[1] Wang JP, Huang D, Chang SC, Wu YM. New evidence and perspective to the Poisson process and earthquake temporal distribution from 55,000 events around Taiwan since 1900. 2014. Nat Hazard Rev ASCE 2014;15:38–47.
[2] Wu CF, Lee WH, Huang HC. Array deployment to observe rotational and translational ground motions along the Meishan fault, Taiwan: a progress report. Bull Seismol Soc Am 2009;99:1468–74.
[3] Wang JP, Lin CW, Taheri H, Chen WS. Impact of fault parameter uncertainties on earthquake recurrence probability by Monte Carlo simulation—an example in central Taiwan. 2012. Eng Geol 2012;126:67–74.
[4] Papaioannou I, Straub D. Reliability updating in geotechnical engineering including spatial variability of soil. Comput Geotech 2012;42:44–51.
[5] Zhang J, Tang WH, Zhang LM, Huang HW. Characterising geotechnical model uncertainty by hybrid Markov Chain Monte Carlo simulation. Comput Geotech 2012;43:23–36.
[6] Ferni G, Mannani G. Bayesian approach for uncertainty quantification in water quality modeling: the influence of prior distribution. J Hydrol 2010;392:31–9.
[7] Esteva L. Seismicity prediction: a Bayesian approach, In: Proceedings of the fourth world conference on earthquake engineering; 1969; p. 172–84.
[8] Cua G., Heaton T. The virtual seismologist (VS) method: a Bayesian approach to earthquake early warning. In: Earthquake early warning systems (Springer), Manfredi G, Zschau J, editors; 2007. p. 97–132.
[9] Lancieri M, Zollo A. A Bayesian approach to the real-time estimation of magnitude from the early P and S wave displacement peaks. J Geophys Res Solid Earth 2008;113:17.
[10] Arnold R, Townend J. A Bayesian approach to estimate tectonic stress from seismological data. Geophys J Int 2007;170:1336–56.
[11] Mignan A, Verner MJ, Wiemer S, Chen CC, Wu YM. Bayesian estimation of the spatially varying completeness magnitude of earthquake catalogs. Bull Seismol Soc Am 2011;101:1371 -1358.

[12] Imoto M, Ishiguro M. A Bayesian approach to the detection of changes in the magnitude-frequency relation of earthquakes. J Phys Earth 1986;5:441–55.
[13] Imoto M. Changes in the magnitude-frequency b-value prior to large ($M>6$) earthquakes in Japan. Tectonophysics 1991;193:311–25.
[14] Wang JP, Brant L. Two Bayesian algorithms for earthquake parameters estimation. Earthquake Spectra 2015;31:1–14.
[15] Ang AHS, Tang WH. Probability concepts in engineering: emphasis on applications to civil and environmental engineering. NJ: John Wiley & Sons, Inc.; 2007.
[16] Lin CW, Lu ST, Shih TS, Lin WH, Liu YC, Chen PT. Active faults of central Taiwan. Special Publ Cent Geol Surv 2008;21:148.
[17] Lin CW, Chen WS, Liu YC, Chen PT. Active faults of eastern and southern Taiwan. Special Publ Cent Geol Surv 2009;23:178.
[18] Wang JP, Kuo-Chen H. On the use of AFOSM to estimate major earthquake probabilities in Taiwan. Nat Hazard 2015;75:2577–87.
[19] Omori F. Preliminary note on the Formosa Earthquake of March 17, 1906. Bull Imp Earthquake Invest Committee 1907;1:53–69.
[20] Shih R, Chen W, Lin C. A new look for the cause of the 1906 Meishan earthquake in southwestern Taiwan from shallow seismic reflection images. In: American geophysical union, fall meeting abstracts; 2003.
[21] Cheng CT, Chiou SJ, Lee CT, Tsai YB. Study on probabilistic seismic hazard maps of Taiwan after Chi–Chi earthquake. J Geoeng 2007;2:19–28.
[22] Wells DL, Coppersmith KJ. New empirical relationships among magnitude, rupture length, rupture width, rupture area, and surface displacement. Bull Seismol Soc Am 1994;84:974–1002.
[23] Anderson JG, Wesnousky SG, Stirling MW. Earthquake size as a function of fault slip rate. Bull Seismol Soc Am 1996;86:683–90.
[24] Gutenberg B, Richter CF. Frequency of earthquake in California. Bull Seismol Soc Am 1944;34:1985–8.
[25] Gardner JK, Knopoff L. Is the sequence of earthquakes in southern California, with aftershocks removed, Poissonian? Bull Seismol Soc Am 1974;64:1363–7.
[26] Wang JP, Brant L, Wu YM, Taheri H. Probability-based PGA estimations using the double-lognormal distribution: including site-specific seismic hazard analysis for four sites in Taiwan. Soil Dyn Earthquake Eng 2012;42:177–83.
[27] Wu YM, Chen CC. Seismic reversal pattern for the 1999 Chi–Chi, Taiwan M_w 7.6 earthquake. Tectonophysics 2007;429:125–32.
[28] Ashtari Jafari M. Statistical prediction for the next great earthquake around Tehran. J Geodyn 2010;49:14–8.
[29] Abramson LW, Lee TS, Sharma S, Boyce GM. Slope stability and stabilization methods. 2nd ed.. NJ: John Wiley & Sons, Inc.; 2002.
[30] Ellsworth WL, Matthews MV, Nadeau RM, Nishenko SP, Reasenberg PA, Simpson RW. A physically based earthquake recurrence model for estimation of long-term earthquake probabilities. Open-file rep. 99-522, USGS, Washington, DC.
[31] Song Y, Gong J, Gao S, Wang D, Cui T, Li Y, et al. Susceptibility assessment of earthquake-induced landslides using Bayesian network: a case study in Beichuan, China. Comput Geosci 2012;42:189–99.
[32] Amighpey M, Voosoghi B, Motagh M. Deformation and fault parameters of the 2005 Qeshm earthquake in Iran revisited: a Bayesian simulated annealing approach applied to the inversion of space geodetic data. Int J Appl Earth Obs Geoinf 2014;26:184–92.
[33] Koutsourelakis PS. Assessing structural vulnerability against earthquakes using multi-dimensional fragility surfaces: a Bayesian framework. Probab Eng Mech 2010;25:49–60.
[34] Vu D, Xue M, Tan X, Li J. A Bayesian approach to SAR imaging. Digit Signal Process 2013;23:852–8.
[35] Hapke C, Plant N. Predicting coastal cliff erosion using a Bayesian probabilistic model. Mar Geol 2010;278:140–9.
[36] Ramin M, Labencki T, Boyd D, Trolle D, Arhonditsis GB. A Bayesian synthesis of predictions from different models for setting water quality criteria. Ecol Model 2012;242:127–45.