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# Earthquake probability in Taipei based on non-local model with limited local observation: Maximum likelihood estimation

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ABSTRACT

Many earthquake empirical models were developed based on the statistics in the past. However, it is commonly seen that a non-local model was applied to a local study without any adjustment. In this paper, a new algorithm using maximum likelihood estimation (MLE) to adjust a non-local model for local applications was presented, including a case study assessing the probability of major earthquake occurrences in Taipei. Specifically, considering the fault length of 36 km and slip rate of 2 mm/yr, it suggests the Sanchiao (or Shanchiao) fault could induce a major earthquake with magnitude $M_w$ 7.14 ± 0.17, based on a non-local model integrated with limited local data using the MLE algorithms.

## 1. Introduction

Given our imperfect understandings and natural randomness, many earthquake empirical models were developed with earthquake statistics of the past [1–6]. For example, Wu and Kanamori proposed a relationship between PD3 and PGV (maximum ground displacement in the first three seconds and peak ground velocity) that became a key empirical relationship to on-site earthquake early warning [1]. Similarly, several empirical models between earthquake magnitude and different fault characteristics (e.g., fault length) were developed [2,3], which are useful to earthquake potential assessment for a mapped fault [7,35]. Moreover, ground motion prediction equations that are essential to seismic hazard analysis are usually an empirical model [4–6]. For instance, Lin et al. developed a series of local ground motion models based on the data in Taiwan [6], which were essential to seismic hazard assessments for the region [8].

From the examples above, we can see that earthquake empirical models play an important role in earthquake study, considering that earthquake analytical models are still difficult to be reliably developed (mainly owing to nature randomness and our imperfect understanding). However, from model development to application, the following question is often asked and encountered: Are non-local empirical models suitable for local applications?

A possible solution to this epistemic uncertainty is to develop a local empirical model from scratch, then applying it to any local applications. However, for an earthquake study, the data are usually very limited owing to the long return period of major earthquakes, making it difficult to develop a local empirical model with a representative sample size. As a matter of fact, in the highly-cited study by Wells and Coppersmith [2], the proposed models were developed with data from several regions based on a more representative sample size, with the presumption that the data belong to the same population regardless of locations.

On the other hand, although the local data are too limited to develop a local model, they should be utilized in a local application considering the higher data relevancy. As a result, for a local application it is logical to integrate local data with non-local models based on a robust algorithm, as maximum likelihood estimation (MLE) that was commonly utilized in different applications and studies under such situation [9–15]. As a result, the motivation of the study is also aiming to use MLE to integrate a non-local model with limited local data, then applying the newly adjusted model to the target problem for evaluating major earthquake probabilities in Taipei, the most important city in Taiwan.

The paper in the following is organized as follows: The geological background of the study area is given in Section 2; an earthquake empirical model between earthquake magnitude, fault length, and slip rate is detailed in Section 3; the overviews of MLE, the algorithm developed, and the model application are given in Section 4, followed by the discussions over several issues, such as earthquake randomness and epistemic uncertainty, that are also related to the study.

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## 2. The Sanchiao fault in Taipei

Taiwan is known for high seismicity owing to its location close to the boundary of three tectonic plates in the west of the Pacific Ocean. On average, there are around 2000 earthquakes above $M_L$ 3.0 occurring in the region, and a catastrophic event, like the $M_w$ 7.6 Chi-Chi earthquake in 1999, could be able to recur in decades [16]. Consequently, a variety of earthquake studies focused on the study region were conducted and reported, from seismic hazard analyses [8], to earthquake early warning [17,18], to earthquake statistics study [19,20], among others [21–26].

Taipei, the most important city in Taiwan, is therefore quite susceptible and vulnerable to earthquake hazard, not to mention the consequence should be more severe if a major earthquake occurs in the city (with a six-million population). As a result, the local community has been continuously studying the potentials of major earthquakes that could recur/occur around the city. For example, by examining the sediment sequences from deep boreholes along the Sanchiao fault in Taipei, the study concluded that the fault should have induced at least three major earthquakes with magnitude around $M_w$ 7.0 in a period of 2600 years in early Holocene [21]. Besides, by studying the so-called neo-tectonic structures in the field, the researchers considered the Sanchiao fault is capable of inducing a major earthquake with magnitude above $M_w$ 6.5 [22]; then with a Bayesian calculation, a study suggested that the return periods of the Sanchiao earthquake could be around 550–750 years [23]. Therefore, from the studies above it should be understood that the Sanchiao fault is the major concern to the (seismic) safety of the city. Fig. 1 shows the location of the fault in north Taiwan, and the geological background of the area.

From engineering perspectives, studies like seismic hazard assessment and earthquake probability evaluation were also reported for the study area. For example, Wang et al. [8] conducted a detailed probabilistic seismic hazard analysis (PSHA) for the city, and suggested 12 earthquake time histories that properly matched the hazard levels for the site's performance-based, earthquake-resistant designs. On the other hand, considering the basin topography could amplify ground shakings, Solokov et al. [24] studied and quantified the basin effect in Taipei by cross checking ground motion records (i.e., time histories) inside and outside the basin. As to earthquake probability evaluation, statistical studies on the earthquake records of the past were also reported, aiming to estimate earthquake probabilities for some preparedness work from the historical data and trend [19,20].

As shown in Fig. 1, the mapped length of the Sanchiao fault in Taipei is commonly considered at 36 km, which have been adopted in several applications [7,23]. In addition, the slip rate of the fault was considered around 2 mm per year [7,23]. For other properties (such as rupture area and displacement) that are not as essential as length and slip rate to this study present herein, refer to the investigation report [23] for more details.

## 3. Empirical relationship between earthquake magnitude, fault length, and slip rate

Mainly based on the data from North America with a sample size of 43, Anderson et al. [3] proposed the following empirical model between earthquake magnitude, fault length and slip rate:

$$M_w = 5.12 + 1.16\log(L) - 0.2\log(S) + e \; ; \quad \sigma_e = 0.26 \tag{1}$$

where $M_w$ is earthquake moment magnitude, $L$ is fault length in km, $S$ is slip rate in mm/yr, and $e$ is error term or model error. Note that the standard deviation of $e$ was characterized as 0.26 from the level of sample scattering, and based on regression theory it is a random variable following the normal distribution with mean value = 0 [27]. Also note that the empirical model showing a positive correlation between earthquake magnitude and fault length should be rational, since a longer active fault should more possibly trigger a more extensive rupture or displacement, leading to more energy release and causing a bigger earthquake. On the other hand, it is also rational that the magnitude should be negatively correlated with slip rate, considering the possibility that a creep movement could release less strain energy over time, then ending up with a more brittle and explosive failure at critical points.

This model developed by Anderson et al. [3] can be considered a continuous work of the highly-cited study (with more than 4500

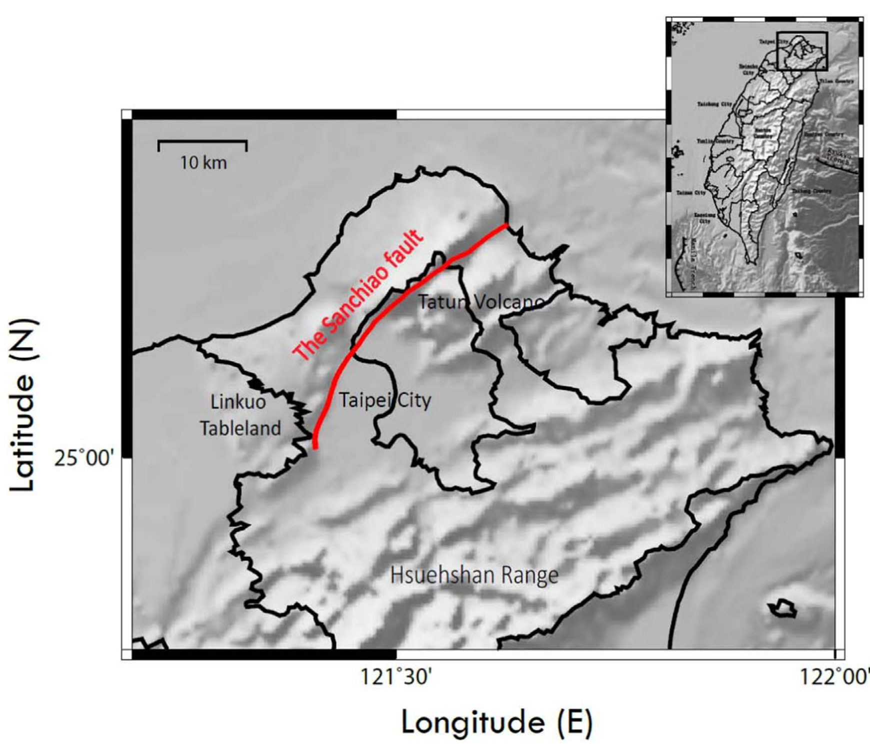


**Fig. 1.** Location of the Sanchiao fault in north Taiwan and the geological background of the study area.

citations as February 2017) by Wells and Coppersmith who proposed several similar relationships based on a very comprehensive review on global (major) earthquake data. On top of the empirical relationships, both studies also provided statistical measures on the models' uncertainty, such as the standard deviation of model error and the coefficient of determination. By comparing those statistical measures, Anderson et al. [3] concluded that the inclusion of slip rate in such regressions for earthquake magnitude predictions could reduce misfits between the observed and predicted values more effectively than those sorely based on fault length. Accordingly, they further suggested the model should be used in future applications for more accurate predictions when slip rate data are available. Considering this, a recent study firstly applying advanced first-order second-moment (AFOSM) calculations to earthquake magnitude predictions [7] was therefore adopting the model as the underlying performance function over others, in an attempt to improve the reliability of the final estimates on the basis of the empirical model with less aleatory uncertainty. Under the same consideration, this study is also based on the Anderson model for earthquake magnitude prediction, which can improve the accuracy of the probability estimates present herein according to Anderson et al. [3].

However, it is noted that this empirical relationship was developed without using any local data from Taiwan. Then as mentioned previously, a logical question will be raised as follows: Could we use the non-local empirical model for the study area in Taipei?

A logical way to evaluate the problem is to compare the non-local model to local data. From the literature [7,23,28], two complete sets of local data in terms of earthquake magnitude, fault length, and slip rate are as follows: a) Case 1: the 1999 $M_w$ 7.6 Chi-Chi earthquake related to the Chelungpu fault with fault length of 90 km and slip rate of 15 mm/yr, or denoted as (7.6 $M_w$, 90 km, 15 mm/yr); and b) Case 2: the 1906 $M_w$ 6.4 Meishan earthquake related to the Meishan fault with fault length of 14 km and slip rate of 6 mm/yr, i.e., (6.4 $M_w$, 14 km, 6 mm/yr). More discussion regarding the data mining is given in Section 5.3.

On the basis of Case 1 (7.6 $M_w$, 90 km, 15 mm/yr), we found that the empirical model (Eq. (1)) would underestimate the earthquake magnitude as $M_w$ 7.15 compared to the observation of $M_w$ 7.6. It must be noted that this "0.45-magnitude" underestimation is by no means insignificant, considering the exponential increments in describing earthquake magnitude (i.e., $M_w$ 7.0 earthquakes will release energy around 30 times as much as $M_w$ 6.0 earthquakes, and so on so forth) [29]. To be more specific, the $M_w$ 7.15 earthquake from the empirical model is actually only 20% as big as the $M_w$ 7.6 earthquake that has occurred and been measured, from the perspective of energy release that could reflect earthquake hazards more linearly.

Based on the data of (6.4 $M_w$, 14 km, 6 mm/yr) from the other case, although the magnitude estimate of $M_w$ 6.29 from the non-local model shows a better agreement with the observation of $M_w$ 6.4, the model still underestimates the observation by 0.11. As a result of that, the goodness-of-fit evaluation seems to show that the non-local model could possibly underestimate earthquake magnitude when used in Taiwan, considering the underestimation was happening in the two local major earthquakes since 1900.

Fig. 2 shows a graph summarizing the model's goodness-of-fit to the local data. From the graph, it is clearer to see the non-local model underestimates the earthquake magnitudes in both events, and therefore the model should be better adjusted before a local application. Specifically, in order to "fix" the issue, a logical and straightforward approach is to adjust the original intercept (i.e., 5.12) to be greater, which is one key, logical presumption in the MLE analysis proposed in the following.

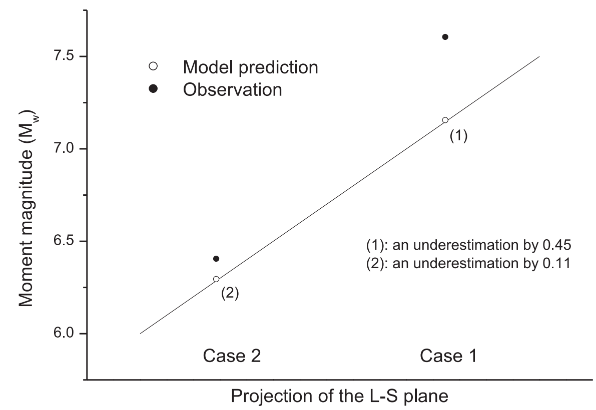


**Fig. 2.** Goodness-of-fit between the non-local model and the two local events and the datasets: Case 1 ($M_w$ 7.6, 90 km, 15 mm/yr) and Case 2 ($M_w$ 6.4, 14 km, 6 mm/yr).

## 4. Maximum likelihood estimation and applications

### 4.1. Overviews

The method of moment and maximum likelihood estimation (MLE) are probably the two most important approaches in statistical estimating [27]. Unlike the method of moment, MLE aims to estimate the statistics (e.g., mean value) of a random variable by maximizing the probability as the given samples that are collected.

We used the following example to further demonstrate this statistical estimation: Let a binomial variable $X$ denote the outcomes of a project as successful (1) and not successful (0); then given the outcomes of three projects as 1, 1, and 0, we first understand that the probability of success, denoted as $p$, should be equal to 0.67 based on the three samples.

Then, using MLE procedures to obtain such a statistical inference is as follows: First, the probability for the given observation (1, 1, 0) to occur can be expressed as $p \times p \times (1 - p) = p^2 \times (1 - p)$, which is also the likelihood function of this MLE. Since $p$ is the unknown, the purpose of MLE is to find $p$ that can maximize the likelihood function. As a result, when $p$ equates to 0.67, the equation or probability $p^2 \times (1 - p)$ can be maximized as 0.15 (=4/27). Therefore, MLE is a statistical estimation in searching for $\theta$ that can maximize the likelihood function $\Pr(\varepsilon|\theta)$, where $\varepsilon$ denotes the observation or samples. As mentioned previously, several MLE-based algorithms have been developed and used in earthquake engineering, such as the MLE method in calibrating the $b$-value of the Gutenberg-Richter relationship [9,11].

### 4.2. Review of regression analysis

Before applying MLE to this study and making the following derivations more understandable, we would like to briefly review the regression analysis that was used to develop the prior, non-local model. For a regression model, its general formulation can be expressed as [27]:

$$Y = f(X) + e \tag{2}$$

where $Y$ and $X$ denote the independent and dependent variables in regression analysis, and $e$ denotes the model error, which is a random variable following the normal distribution with mean value = 0. (Note that model error is usually denoted as $\varepsilon$ in regression analysis, but in order to avoid confusion from the observation $\varepsilon$ also used in MLE algorithms, we used $e$ instead of $\varepsilon$ herein).

Understandably, from the regression model, $Y_x$, the predicted $Y$ value given $X = x$, is equal to $f(x) + e$, where $f(x)$ is a constant as $x$, while model error $e$ is a random variable by definition. Therefore, the mean value (denoted as $E$) and variance (denoted as $V$) of $Y_x$ can be derived and expressed as follows [27]:

$$E[Y_x] = E[f(x) + e] = E[f(x)] + E[e] = E[f(x)] \quad (3)$$

$$V[Y_x] = V[f(x) + e] = V[e] \quad (4)$$

Explicitly, the derivation in Eq. (3) is on the basis that the mean value of $e$ in a regression model is equal to zero, and Eq. (4) is based on the theory of probability that the variance of a constant is zero [27]. Note that variance is the square of standard deviation.

As mentioned previously, model error $e$ follows the normal distribution according to the fundamentals of regression analysis, and therefore $Y_x$ is also a variable following the normal distribution as $e$ does, just like $C = D + 1010$ in which $C$ and $D$ must follow the same probability distribution regardless.

Note that although the reviews on regression analysis are on the basis of a single regression model $Y = f(X) + e$, such basics and derivations are completely applicable to a multiple regression model $Y = f(X_i s) + e$, like the "Anderson" relationship we used in the study governed by two independent variables (fault length and slip rate) as indicators.

### 4.3. The MLE-based model adjustment

This section would like to elaborate the new MLE algorithm for adjusting the non-local (empirical) model with (limited) local data, then using the adjusted model in the local application to assess the major earthquake probability in Taipei. As mentioned previously, the intercept should be larger for the study region, since the model underestimated the two local events (see Fig. 2).

Besides, an additional finding from the goodness-of-fit assessment is that the model uncertainty might not be as large as 0.26 in association with the non-local model, considering the "local" standard deviation should be closer to 0.18 based on the two events with respective model differences as $M_w$ 0.45 (7.6 vs 7.15) and $M_w$ 0.11 (6.4 vs 6.29). In other words, to better reflect the situations also observed in the goodness-of-fit evaluation, the model error of such a correlation around Taiwan should be adjusted to be lower simultaneously.

On the basis of the two findings from the goodness-of-fit assessment, the likelihood function of the new MLE algorithm can be expressed as follows:

$$\Pr(\varepsilon{:}\ Case\ 1\ and\ Case\ 2 | A = a;\ \ \sigma = \sigma^*) \quad (5)$$

where $A$ and $\sigma$ denote the intercept and the standard deviation of model error, and $a$ and $\sigma^*$ are the adjusted parameters subject to the observations from Case 1 and Case 2. Note that $A$ and $\sigma$ are both symbols in the derivation, and $a$ and $\sigma^*$ denote constants.

Next, by substituting $a$ and $\sigma^*$ into Eq. (1), the adjusted model becomes:

$$M_w = a + 1.16\log(L) - 0.2\log(S) \pm \sigma^* \quad (6)$$

Consequently, for Case 1 with $L = 90$ km and $S = 15$ mm/yr, the mean magnitude is then equal to $(a + 2.03)$ from the adjusted model, with a standard deviation equal to $\sigma^*$ as explained in Section 4.2.

As a result, the probability for the observation of $M_w$ 7.6 to occur can be expressed as Eq. (7), subject to mean magnitude $= (a + 2.03)$ and standard deviation $= \sigma^*$:

$$\Pr(M_w = 7.6 | \mu = a + 2.03;\ \sigma = \sigma^*) \quad (7)$$

Then considering the target variable ($M_w$) as the dependent variable of a regression model following the normal distribution (its probability density function is given in the Appendix A), Eq. (7) can be extended as:

$$\Pr(M_w = 7.6 \mid \mu = a + 2.03;\ \sigma = \sigma^*) = \frac{1}{\sigma^*\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{7.6 - (a + 2.03)}{\sigma^*}\right)^2\right] \quad (8)$$

With the function governed by two unknowns $a$ and $\sigma^*$, Eq. (7) or Eq. (8) were rewritten as:

$$\Pr(M_w = 7.6 \mid \mu = a + 2.03;\ \sigma = \sigma^*) = g_1(a, \sigma^*) \quad (9)$$

Similarly, we can exercise the same derivation for Case 2 with data as (6.4 $M_w$, 14 km, $S = 6$ mm/yr), and wrote its likelihood function as $g_2(a, \sigma^*)$. Finally, because the two local samples are the observation as a whole, the likelihood function of this MLE becomes:

$$\Pr(\varepsilon{:}\ Case\ 1\ and\ Case\ 2 \mid A = a;\ \ \sigma = \sigma^*) = g_1(a, \sigma^*) \times g_2(a, \sigma^*) \quad (10)$$

With the likelihood function developed, the MLE-based model adjustment is nearly finished. The final step is to solve $a$ and $\sigma^*$ that can maximize the likelihood function in this MLE calculation.

### 4.4. The adjusted model

With the two local datasets as (7.6 $M_w$, 90 km, 15 mm/yr) and (6.4 $M_w$, 14 km, 6 mm/yr), we solved the governing equation or the likelihood function in Eq. (10), and returned $a$ as 5.4 and $\sigma^*$ as 0.17 that can maximize the probability from the governing equation. Therefore, the new adjusted model we proposed for local applications in Taiwan is:

$$M_w = 5.4 + 1.16\log(L) - 0.2\log(S) \pm 0.17 \quad (11)$$

It is worth noting that the adjusted model does not conflict with the presumptions set up at the beginning from the goodness-of-fit assessments; that is, the intercept should be adjusted to be greater (5.4 vs 5.12) while the standard deviation to be lower (0.17 vs 0.26).

Fig. 3 shows the adjusted model and its goodness-of-fit to the same local data. That is, unlike the original model that underestimates both local events, the adjusted model is more random with one underestimation and one overestimation, which are by 0.17 for Case 1 (underestimation) and −0.17 for Case 2 (overestimation), respectively.

On the basis of the adjusted model, Fig. 4 shows the probability density for each case in detail. The calculation shows that the probability density is 1.41 for both cases (although in different sides), leading to the likelihood function of this MLE equal to 2.0 ($= 1.41 \times 1.41$) with new parameters of $a = 5.4$ and $\sigma^* = 0.17$. It must be noted that since the earthquake magnitude is a continuous random variable, the probability density is allowed to be greater than 1.0.

### 4.5. The application of the adjusted model to the target problem

From the goodness-of-fit assessments showing the non-local model could probably lead to underestimation when used locally in Taiwan, to the use of MLE algorithms to adjust the model with limited local data, the next key task is to apply the adjusted model to the target problem of this study: the earthquake magnitude that could be possibly induced by the Sanchiao fault in Taipei. Specifically, with the Sanchiao fault characterized as 36-km long with 2-mm slip rate per year, the best-

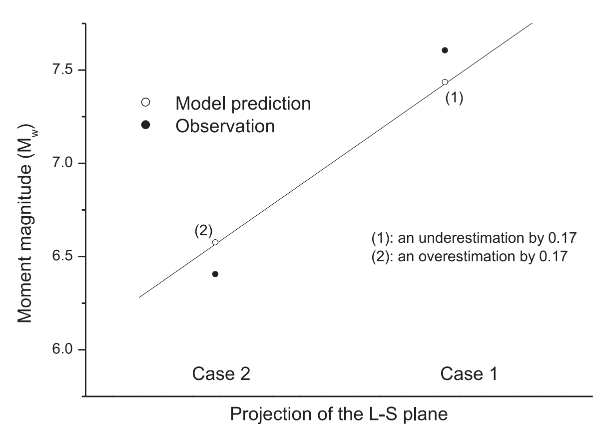


**Fig. 3.** Goodness-of-fit between the newly-adjusted, MLE model and the same two local events shown/used in Fig. 2.

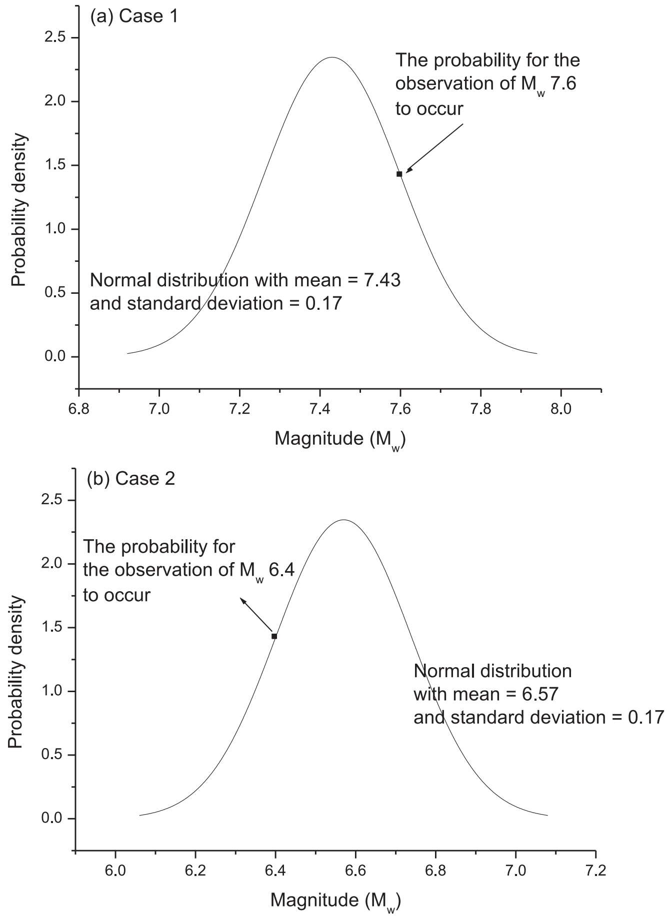


**Fig. 4.** The probability densities for each of the two local events to occur on the basis of the MLE model.

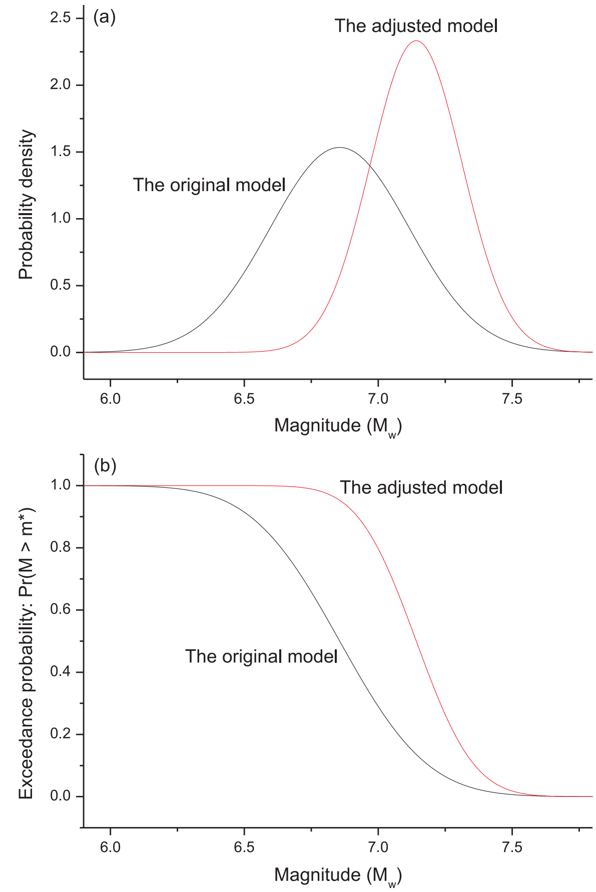


**Fig. 5.** Comparison on the probability estimates based on the non-local model and the adjusted model towards the earthquake magnitude induced by the Sanchiao fault in Taipei: a) probability density functions, and b) exceedance probability.

estimate earthquake magnitude or the ideal statistical inference based on this robust, transparent MLE algorithm was calculated as $M_w$ 7.14, which is a result of integrating the non-local model with local data in such a unique manner and calculation.

However, it must be noted that the best estimate (i.e., $M_w$ 7.14) present in one single value is the so-called deterministic estimate. By contrast, a probabilistic estimate on the target problem would become $M_w$ 7.14 ± 0.17 when the model error was also taken into account. By comparison, the non-local model, developed without considering any local data, would suggest a deterministic and probabilistic estimate as $M_w$ 6.87 and $M_w$ 6.87 ± 0.26, respectively, lower than those based on the adjusted model present herein.

Given the target variable ($M_w$) in a regression model following the normal distribution, Fig. 5 shows the earthquake magnitude probability functions related to the Sanchiao fault. For example, the probabilities for the fault to induce a major earthquake above $M_w$ 7.0 are 80% and 30% based on the adjusted and original models, respectively. By contrast, the probabilities for the fault to induce a catastrophic event like the 1999 $M_w$ 7.6 Chi-Chi earthquake in central Taiwan are 0.37% (adjusted model) and 0.23% (non-local model). From the estimations, we can see that the adjusted model suggests a larger earthquake probability for the target problem than the original model does, which is reasonable and expectable given the purpose of the MLE analysis is to address the underestimations that have been happening in the two local events, as shown in Fig. 2.

## 5. Discussions

### 5.1. Earthquake randomness

Understandably, earthquake randomness would appear in magnitude, as well as in location, recurrence interval, etc. But it is worth noting that the scope of this study aims to estimate the earthquake magnitude in association with the Sanchiao fault based on a MLE-based algorithm integrating or adjusting a non-local empirical model with limited local data. In other words, analyzing and calculating when the event could occur is not the scope of the study, although it is also relevant to the target problem.

Nevertheless, the new estimate on the magnitude present herein can be further integrated with the "time uncertainty" regarding the Sanchiao earthquake. For example, given a best-estimate return period of the event is around 600 years [23], the probability for it to recur within the next 50 years should be around 8%, based on the Poisson model that is commonly accepted for such temporal probability evaluations [30,31]. As a result, the probability for the Sanchiao earthquake with magnitude above $M_w$ 7.6 in next 50 years might be around 0.03%, considering "time uncertainty" and "size uncertainty" simultaneously.

### 5.2. Epistemic uncertainty

From the MLE calculation above, another technical question could be raised as follows: Why not adjust the four model parameters altogether, but only focusing on the intercept and the standard deviation of model error? The response to the question is simple: if the coefficients are adjusted simultaneously, the calculation is then a "pure" regression analysis based on the local data only, without any considerations of the prior information. On the other hand, since the minimum sample size to perform such a multiple regression is three, the local regression cannot be developed with the two samples anyway.

However, one may adopt a similar MLE calculation to only adjust the intercept, on the consideration that the model error should be more or less the same in different regions. Admittedly, even though we found the model error for the study region seems smaller based on the goodness-of-fit assessment, we cannot perfectly prove, and no one can, such a presumption is less logical/efficient than ours. As a result, this is the so-called epistemic uncertainty due to our imperfect understandings of a problem [32] that cannot be perfectly proven at this moment. Similarly, it is an epistemic uncertainty as we kept the two other coefficients unchanged in the MLE calculation, considering the "slopes" between magnitude and length and between magnitude and slip rate should be similar in different regions. Most importantly, although no one can perfect address the epistemic uncertainties, we consider the inferences from this MLE calculation combining the two sources of information (i.e., global model and limited local data) in such a unique way should be more logical and reliable than those sorely based on the prior model without any consideration of local situations/data, with the support that the newly adjusted model renders a more random prediction, unlike the prior non-local model that always underestimates earthquake magnitudes in the two local events.

### 5.3. Other relevant issues

Although the region around Taiwan is known for high seismicity, there are only two (local) major events complete with earthquake magnitude, fault length, and slip rate. Note that it is acknowledged that several studies did provide their best-estimate magnitudes associated with other active faults in Taiwan from indirect evidence, but they are not as reliable as those we used in this study based on direct instrumentations and measurements. Therefore, for reducing the uncertainties, the data from direct measurements were only used in this study, which leads to limited samples available to this study.

On the other hand, what if the local samples are adequate (say 100) for developing a representative local relationship, what would we do under the situation? Admittedly, in this situation we have to agree that this MLE model will become redundant, and using the "100%" local model developed with local data should be a more logical and acceptable choice. The reason is that the local data are always more relevant and reliable than non-local data, which is a theory that is more acceptable to most of us although it is still debatable owing to our imperfect understandings, or epistemic uncertainty.

In fact, there are some basic principles to properly integrate prior information with limited local observation to obtain a new inference [33,34]. That is, the reason of considering the less reliable non-local data in a local study is owing to the lack of adequate local data. On the contrary, when local samples are adequate, adding prior data to the analysis becomes redundant, even making the estimate more questionable. Here is a simple analogy to better explain this: If one wants to study the relationships between heart attack and age among Taiwanese people, the inference will be most reliable based on (many) local data; the inference based on non-local data and limited local data will be the second best, while the one sorely based on non-local data will be least favorable.

## 6. Conclusion

Taipei, the most important city in Taiwan, is vulnerable to earthquake hazard owing to the unique geological background around the region. Therefore, the target problem of the study is to evaluate the major earthquake that could be potentially induced by the Sanchiao fault in Taipei. On the basis of a non-local empirical model and limited local samples, this paper presents a novel MLE algorithm aiming to adjust the non-local model for the local application. Specifically, with the adjusted model from MLE, the inference is that the magnitude of the recurring Sanchiao earthquake should be $M_w$ 7.14 ± 0.17, considering the fault length of 36 km and slip rate of 2 mm/yr. That is, the inference based on both the non-local model and local data would suggest a 0.37% probability for the Sanchiao fault in Taipei to induce a catastrophic event as disastrous as the 1999 $M_w$ 7.6 Chi-Chi earthquake striking central Taiwan.

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## Appendix A. The probability density function of the normal distribution

The following equation is the probability density function of the normal distribution [27]:

$$f(X = x; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right] \quad \text{(A.1)}$$

where μ and $\sigma$ are the mean and standard deviation of a random variable $X$ following the normal distribution.

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